University of Ruhuna

Bachelor of Science General Degree Level I
(First Semester Examination)

June/July 2015

Subject: Mathematics
Course Unit: MAT112δ/MAM112α(Differential Equations)

Time: One (01) Hours

Answer 02 Questions only.

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1. a) Explain how you would obtain the solution of a differential equation of the form $\dfrac{dy}{dx} = \dfrac{ax+by+c}{a_1x+b_1y+c_1}$; where a, b, c, a_1, b_1, c_1 are constants.
Solve the following differential equation:
$\dfrac{dy}{dx} = \dfrac{x+2y-3}{2x+y-3}$.

b) If $n \neq 0$ and $n \neq 1$ explain how you would solve a differential equation of the form $\dfrac{dy}{dx} + P(x)y = Q(x)y^n$; where $P(x)$ and $Q(x)$ are functions of x only.
Solve the differential equation
$x\dfrac{dy}{dx} + y = y^2 \ln x$.

c) Find the general and singular solution of the differential equation
$y = px + p - p^2$; where $p = \dfrac{dy}{dx}$.

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2. a) State a necessary and sufficient condition that should be satisfied by $M(x,y)$ and $N(x,y)$ so that the differential equation $M(x,y)dx + N(x,y)dy = 0$ is exact.

Show that the differential equation
$\left(\cos x \ln(2y-8) + \dfrac{1}{x}\right) dx + \left(\dfrac{\sin x}{y-4}\right) dy = 0$ is exact and hence solve it.

b) Obtain a suitable integrating factor for the differential equation $(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$ to be exact and hence solve it.

c) Find the Orthogonal trajectories of the family of curves $x^2 - 2\lambda x + y^2 = 4$, where λ is a parameter.

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3. a) If $F(D) \equiv a_0 D^n + a_1 D^{n-1} + ... + a_n$ and a is a non-zero constant,prove that

(i) $\frac{1}{F(D)}\{e^{ax}V(x)\} = e^{ax}\frac{1}{F(D+a)}\{V(x)\}$ and

(ii) $\frac{1}{F(D^2)}\{\cos ax\} = \frac{1}{F(-a^2)}\{\cos ax\}$ if $F(-a^2) \neq 0$; where $D \equiv \frac{d}{dx}$.

Solve the following differential equation:

$$\frac{d^2y}{dx^2} + y = \sin x \sin 2x.$$

b) Solve the simultaneous differential equations

$$\frac{d^2x}{dt^2} - 3x - y = e^t$$

$$\frac{dy}{dt} - 2x = 0$$
