University of Ruhuna

Bachelor of Science General Degree
Level I (semester I) Examination – June 2015

Subject: Mathematics
Course Unit: MAT1136/MMS111α (Introductory Statistics)
Time: One (01) hour

Answer 02 questions only
(Calculators and Normal probability charts should be provided)

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(1) (a) State the Baye's Theorem.

Soma is getting married tomorrow, at an outdoor ceremony in a very dry area. In recent years, it has rained only 10 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Soma's wedding?

(b) Let X be a discrete random variable with the following probability mass function:

$$P_X(x=k)=\begin{cases}0.1 & \text{if } k=0\\ 2p & \text{if } k=1\\ 0.3 & \text{if } k=2\\ p & \text{if } k=3\\ 0 & \text{otherwise.}\end{cases}$$

Find
(i) the constant p.
(ii) E(X).
(iii) Var(X).
(iv) If $Y=(X-2)^2$ then E(Y)

(c) Let X be a continuous random variable with the following probability density function:

$$f_X(x)=\begin{cases}cx^3 & 0<x\le 1\\ 0 & \text{otherwise}\end{cases}$$

Find
(i) the constant c.
(ii) Pr(X≥1/2)
(iii) $\Pr(X \ge \frac{2}{3} \mid X > \frac{1}{3})$.

(2) (a) A manufacturer of metal bolts finds that on the average, 20% of his bolts are rejected because they are either oversize or undersize. If he send them to the market as packs of 10 bolts , find the probability that a pack will contain

(i) no more than 2 rejects
(ii) at least 2 rejects

and also find

(iii) the average number of rejects in a pack.

(b) Define the moment generating function of a continuous random variable.

Write down the probability distribution function for an Exponentially Distributed random variable with the parameter α.

Find the moment generating function of the above distribution and **hence** find the expected value and the variance of the variable.

(3).(a) The annual salaries of employees in a large company are approximately normally distributed with a mean of Rs. 25,000 and a standard deviation of Rs. 10,000.

(i) What percent of employees earn less than Rs. 20,000?

(ii) What percent of employees earn between Rs. 22,500 and Rs. 32,500?

(iii) What percent of employees earn more than Rs. 35,000?

(iv) What is the least salary of the top 20%?

(b) The annual number of earthquakes registering at least 2.5 on the Richter scale in South Asia follows a Poisson distribution with mean 12.

Use the Normal approximation to Poisson distribution, find the probabilities that

(i) at least 18
(ii) between 6 and 18
(iii) less than 12

such earthquakes will strike next year?

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