University of Ruhuna - Faculty of Science

Bachelor of Science General Degree - Level I
(Semester I) Examination - July 2015

Subject: Mathematics
Course Unit: MAT1142 / MMA1b30 (Mathematics for Bio Science students)
Time: Two (02) Hours

Answer Four (04) questions only. Calculators will be provided

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1. (a) Let z be a complex number of the form $x+iy$, where x, y are real numbers and i is the imaginary unit.

(i) Write down the complex conjugate $\bar{z}$ of z.

(ii) Show that $z\bar{z}$ and $z+\bar{z}$ are always real.

Write $\dfrac{2+3i}{3-2i}$ in the form $x+iy$, where x and y are to be determined.

(b) Using the binomial expansion, show that

$$(x+\frac{2}{x})^5 = x^5+10x^3+40x+\frac{80}{x}+\frac{80}{x^3}+\frac{32}{x^5}.$$

(c) The number of bacteria N present in a sample is given by $N = 800\,e^{-0.25t}$, where time t is in seconds. Find

(i) the number of bacteria at $t=0$ and

(ii) the time when the number of bacteria reaches 100.

(d) Write down the formulae for $\sin(A+B)$ and $\cos(A+B)$.
Hence, obtain expressions for $\sin 2A$ and $\cos 2A$ in terms of $\sin A$ and $\cos A$.
Using the above expressions you obtained, show that

$$\cot 2A = \frac{\cot^2 A - 1}{2\cot A}$$

and find $\cot 2A$ when $\cos A = \dfrac{1}{2}$ and $\sin A = \dfrac{\sqrt{3}}{2}$.

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2. (a) Find the following limits.

(i) $\lim_{a\to 0} \dfrac{(a+4)^3-64}{a}$.

(ii) $\lim_{x\to 2} \dfrac{x^2-5x+6}{x-3}$

(iii) $\lim_{x\to\infty} \dfrac{5x}{x+1}$

(iv) $\lim_{x\to 3} \dfrac{x^2-9}{x^2-3x}$

Continued.

(b) Differentiate the function $y = x^2 - 2x$ with respect to x using the first principles.

(c) Find the first derivative of each function given below.

(i) $y = x^3 + \sqrt{x} + \frac{3}{x} - 1$

(ii) $y = \sin 3x \cos 3x$

(iii) $y = e^{5x} \ln 5x$

(iv) $y = \frac{2-x}{1+3x}$

(d) The curve represented by the function $y = 2x^3 + ax^2 + bx + c$ has two turning points at $x = 1/2$ and $x = -1$.

(i) Given that the point $(-2, 2)$ is also on this curve, find the values of a, b and c.

(ii) Identify the above turning points as maxima or minima using the second derivative test.

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3. (a) Obtain the first partial derivatives of each function given below with respect to x and y.

(i) $f(x,y) = x^5y^2 + 9x^3y^4 + x^2y^6 + 3x$

(ii) $f(x,y) = x^2e^y + y^3 \ln x$

(iii) $f(x,y) = x\cos y + y\sin x$

(iv) $f(x,y) = y\frac{\ln x}{x}$

(b) If $g(x,y) = 4x^2 - 8xy^4 + 7y^5 - 3$, show that

$$\frac{\partial^2 g(x,y)}{\partial x \partial y} = \frac{\partial^2 g(x,y)}{\partial y \partial x}.$$

(c) A three variable function is given by $h(p,q,r) = p^3q^2r + p^2q + 3pr + 5$.

(i) Find the partial derivatives

$$\frac{\partial h}{\partial p}, \frac{\partial h}{\partial q}, \frac{\partial h}{\partial r}.$$

(ii) Show that the total differential of h at the point $(1,2,1)$ is given by $dh = 19dp + 5dq + 7dr$.

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4. (a) Evaluate the following indefinite integrals.

(i) $\int (2x^4 + \frac{5}{x^3} + 7\sqrt{x} + 5)\, dx$

(ii) $\int (2+5x)^7\, dx$

(iii) $\int (e^{2x} + \sin 2x)\, dx$

(iv) $\int \frac{3x}{x^2+5}\, dx$

(b) Using an appropriate substitution, show that

$$\int \frac{1}{\sqrt{x}+1}\, dx = 2\sqrt{x} - 2\ln(\sqrt{x}+1) + C,$$

where C is an arbitrary constant.

Continued.

(c) Use integration by parts to evaluate the following integrals.

(i) $\int \ln x \, dx$

(ii) $\int x \sin x \, dx$

(d) Find the constants A and B such that

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

Hence, evaluate

$$\int \frac{1}{(x-1)(x+2)} \, dx.$$

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5. (a) (i) Show that

$$\frac{1}{2.303} \int_{100}^{1000} \frac{1}{c} \, dc = 1.$$

(Hint : You may use that $\frac{1}{\log_{10} e} = 2.303$, if necessary.)

(ii) Using partial fractions, evaluate

$$\int_4^6 \frac{1}{x^2 - 9} \, dx.$$

(b) Show that the solution of the differential equation

$$\tan x \frac{dy}{dx} = y; \text{ where } y = 3 \text{ when } x = \frac{\pi}{4},$$

can be written in the form $y = 3\sqrt{2}\sin x$.

(c) Given that K is a constant and

$$\frac{dx}{dt} = K(1-x)^3; \text{ where } x = 0 \text{ when } t = 0,$$

show that $(1-x)^{-2} = 2Kt + 1$.

(Hint : Use the method of separation of variables for (b) and (c) above.)

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6. (a) Classify the following variables as discrete or continuous.

(i) Red blood cell count in the human body

(ii) Phosphorous content of leaves

(iii) Heart rate of a living rat

(iv) Life time of a CFL bulb

(v) Yield of latex per acre

(b) In a continuously assessed course unit, the marks on three different parts carry 20%, 35% and 45% of the total marks respectively. A student scored 65%, 45% and 55% on these parts respectively. Calculate the final mark of the student.

(c) Marks obtained by 20 students in an examination are given below.

60, 83, 71, 81, 74, 56, 64, 72, 54, 84, 63, 50, 83, 60, 83, 52, 80, 84, 71, 75

Find the mean ($\bar{x}$) of this data set.

Construct a table with three columns having x_i, $(x_i - \bar{x})$ and $(x_i - \bar{x})^2$ respectively and hence find the sample variance (s^2), and the standard deviation (s) for this data set.
