University of Ruhuna

Bachelor of Science General Degree
(Level II)Semester I Examination -June/July 2015

Subject : Applied Mathematics/Industrial Mathematics
Course unit : AMT211β/IMT211β/MAM2113 (Fluid Dynamics)

Time :Two (02) Hours

Answer 04 Questions only.

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1. In the usual notation, obtain the equation of continuity in the form $\dfrac{\partial \rho}{\partial t} + div(\rho \underline{q}) = 0$ for a moving fluid.
Deduce that $div\underline{q} = 0$ if the fluid is incompressible.
Show that the motion given by $\underline{q} = \dfrac{k^2(-y\underline{i} + x\underline{j})}{x^2 + y^2}$; where k is a constant, is a possible motion for an incompressible fluid.
Determine the equation of the relevant stream lines. Also test whether this motion is of the potential kind and if so find the velocity potential.

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2. a) The Euler equation of a moving fluid is given by $\underline{F} - \dfrac{1}{\rho}\underline{\nabla} P = \dfrac{D\underline{q}}{Dt}$; where $\underline{F}$ is the external force per unit mass, P is the pressure, ρ is the density of the fluid and $\underline{q}$ is the velocity of the fluid particle.
If the external force is conservative, obtain the Bernoulli equation for an irrotational motion of an incompressible fluid by using above equation.

b) Stationary infinite circular cylinder with radius a placed vertically in a uniform stream of incompressible irrotational fluid, for which undisturbed velocity $-v\underline{i}$; where v is a constant and $\underline{i}$ is a unit vector in the direction OX, which is perpendicular to the axis of the cylinder. Let P be a point in the fluid with $OP = r$, where $r \geq a$ and O lies on the axis of the cylinder. Find

(i) the velocity potential at any point in the fluid and
(ii) the velocity at any point on the cylinder.

Show that the pressure will be positive every where on the surface of the cylinder if $P_\infty > \dfrac{3\rho v^2}{2}$; where $P_\infty > 0$ is the pressure at infinity.

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3. An incompressibe inviscid fluid of uniform density ρ is in an irrotational motion. Show that the kinetic energy T of the fluid enclosed by a surface S is given by $T = -\frac{1}{2}\rho \int_S \phi \frac{\partial \phi}{\partial n} ds$; where ϕ is the velocity potential, $\underline{n}$ is the inward unit normal vector to the surface element ds of S and the fluid is at rest at infinity.
Find the values of A and B so that $(Ar + \frac{B}{r^2})cos\theta$ represents the velocity potential of the motion of an incompressible fluid, which fills the space between a solid sphere of radius b and a concentric spherical shape of radius $a(>> b)$, when the sphere has a velocity $\underline{u}$ and the shell is at rest.
Prove that the kinetic energy of the fluid is $\frac{\pi \rho b^3 u^2 (2b^3 + a^3)}{3(a^3 - b^3)}$; where ρ is the density of the fluid.
Discuss the case when $a \to \infty$.

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4. A simple source of strength m is placed at the origin in an irrotational flow moving with velocity $\underline{u}$ in the direction of OX axis. Show that the velocity potential at any point P in the flow is $\frac{m}{r} - urcos\theta$; where $r = OP$ and θ is the angle XOP.
Deduce that the velocity at P in the plane XOP has components $(\frac{m}{r^2} + ucos\theta, -usin\theta)$, respectively along and at right angle to OP.
Hence show that the stream lines are given by
$r\frac{dr}{d\theta} + r^2 cot\theta = \frac{-m}{usin\theta}$. and the stream lines lie on the surface $ur^2 sin^2\theta - 2mcos\theta =$ constant.

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5. A two dimensional doublet of strength $\mu\underline{i}$ is placed at the point $z = ia$ in a stream of velocity $-V\underline{i}$ in a semi infinite liquid of constant density occupying the half-plane $y > 0$ and having $y = 0$ as a rigid boundary. Show that the complex potential of the motion is $W(z) = Vz + \frac{2\mu z}{z^2 + a^2}$.
Show also that, for $0 < \mu < 4a^2V$, there are no stagnation points on the boundary, and that the pressure on it is minimum at the origin and maximum at the points $x = \pm a\sqrt{3}$.

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6. State the Milne Thomas Circle theorem and its extension.
A source and a sink of equal strength m are placed at the points $(a, 0)$ and $(-a, 0)$ respectively within a fixed circular boundary $|Z| = 2a$. Show that the stream lines are given by
$16a^2y^2 + \lambda y(r^2 - 4a^2) = (r^2 - 16a^2)(r^2 - a^2)$; where λ is a constant and that the fluid speed at the point $(2a, \theta)$ is $\frac{20msin\theta}{a(17 - 8cos2\theta)}$.

Last Page