University of Ruhuna

Bachelor of Science General Degree
Level I (Semester I) Examination - July 2016

Subject: Applied Mathematics
Course Unit: AMT 112β(Mathematical Foundation of Computer Science)

Time :Two (02) Hours

Answer 04 Questions only.

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1. a) Explain what is meant by a tautology and a contradiction.
Using the truth tables determine wether the followings are tautologies, contradictions, or neither:

(i) $(p \wedge q) \wedge (p \rightarrow \sim q)$

(ii) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

b) Show that

(i) $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$

(ii) $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$

logically equivalent or not.

c) Define the inverse, converse and contrapositive of the implication $p \rightarrow q$.
State the inverse, converse and the contrapositive of the satatement:

"If the rain falls, cricket match will be suspend."

d) Symbolize and test the following argument for validity by using a truth table.

"If I join the picnic I can get a better knowledge about the wild life. If I get a better knowledge about the wild life, I will write a book about wild animals. I will not join the picnic. Therefore if I do not join the picnic, I will not write a book about wild animals."

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2. a) Explain what is meant by saying an argument is valid.

Test for the validity of the following using pattern proof:

If I follow the correct dancing steps then I can perform well. If I perform well, I will be able to win the dancing contest. If I do not follow the correct dancing steps, then I will be categorize under the lower points. Therefore, If I do not categorized under lower points, I will be able to win the dancing contest.

b) Consider the following sentences:

All babies are innocent.
Anyone who is innocent and affectionate will be loved by others.
Anyone who is loved by others, will receive gifts.
Teena is an affectionate baby.

(i) Represent the above facts in axioms forms.
(ii) Translate the above axioms into clausal form.
(iii) Use the clausal form to prove that "Teena will receive gifts."

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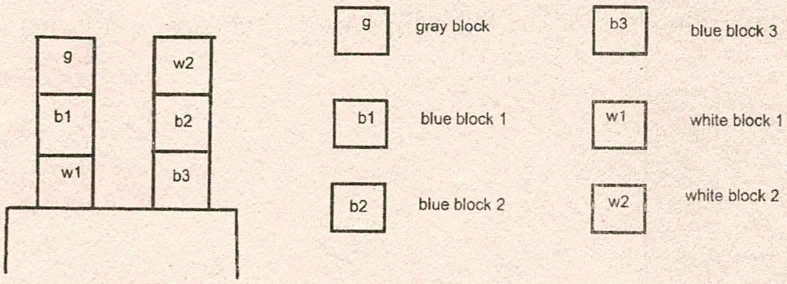
3. a) Consider the following picture, which shows colored blocks stacked on a table. The



following are statements in Prolog that describes this picture.

isabove(g, b_2)	color(g,gray)
isabove(b_1, w_1)	color$(b_1$,blue)
isabove(w_2, b_2)	color$(b_2$,blue)
isabove(b_2, b_3)	color$(b_3$,blue)
	color$(w_1$,white)
	color$(w_2$,white)

The statement "isabove(g, b_1)" and "color(g,gray)" are to be interpreted as "g is above b_1" and "g is colored gray."
Write the answers Prolog whould give if the following questions were added to the program above.

(i) $?color(b_1, blue)$
(ii) $?isabove(X, w_1)$
(iii) $?color(X, blue)$
(iv) $?isabove(b_2, w_1)$

Continued.

b) You are given the following Prolog database in which $child\_of(X, Y)$ asserts that X is a child of Y among individuals in a population.
$child\_of(john, ann)$
$child\_of(john, alex)$
$child\_of(bob, john)$

(i) What would be the answer to the following query.
$?chid\_of(X, alex)$

(ii) Write prolog rules for the followings.

a) If X is a child of Z and Z is a child of Y then X is the $grand\_child\_of$ Y.

b) If Y is a $child\_of$ X then X is a $parent\_of$ Y.

c) If X is a $parent\_of$ Y and X is_female then X is the $mother\_of$ Y.

d) If X is a child of Z and Y is a child of Z then X and Y are siblings.

Now explain how to evaluate the following:
$?grand\_child\_of(A, B)$

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4. a) (i) Represent the decimal number 543.876_{10} in the B.C.D(Binary Coded Decimal) format.

(ii) Convert 000100100011.010100000100 in B.C.D format to the equivalent decimal number.

b) Simplify, $(1011011.01) \div (101.1)$ to 4 decimal places.

c) Convert the following hexadecimal number into binary number.

$$A2F_{16}$$

d) A computer has a word length of 8 bits and uses two's complemenst method for calculations. Translate -102 into the number format used by the above computer.

e) Expalin the method of simplifying $8 - 14$ using two's complements in an 8-bit word length computer.

f) Explain the method of performing the calculation $(103_{10})/(13_{10})$ in a computer with 8-bit word legth that uses two's complements.

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5. a) Define the dual of a proposition concerning a Boolean Algebra $\mathbb{B}$.

Write down the dual of the Boolean expression

$$\bar{y} + (x.y.z).(\bar{x}.\bar{y}.\bar{z}) = 1$$

b) Define what is meant by.

(i) a sum-of-product expression

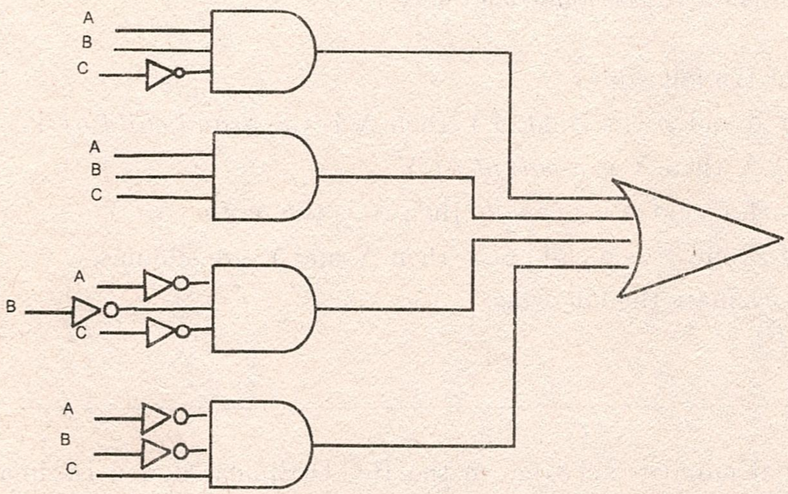
(ii) a complete-sum-of-product expression.

Find the sum-of-product of the expression

$$E = [x + (y(z + x'))'].$$

Hence find the complete-sum-of-products form of the expression.

c) (i) Write down the Boolean expression for the following logic circuit.



(ii) Use a Karnaugh map to find a minimal sum-of-products expression for the network.

(iii) Sketch the logic circuit for the minimal sum-of-products expression you obtained in (ii) above.

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6. a) Use the expand, guess and verify method to show that the closed-form-solution of the recurrence relation

$$S(n) = 3 * S(n-1) \qquad for \qquad n \geqslant 2$$

with the base value $S(1) = 3$ is given by

$$S(n) = 3^n.$$

b) Show that the solution to the linear first-order recurrence relation of the form

$$S(n) = cS(n-1) + g(n)$$

with constant coefficient and the base value S(1) is given by

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$$

The first four numbers of a sequence ae given by 1,6,13 and 22.
Find a recurrence formula of the form $S(n) = S(n-1) + g(n)$ for n^{th} number in the sequence.
Hence find a formula for the n^{th} number in the sequence.
