

University of Ruhuna
Bachelor of Science General Degree
Level I (Semester I) Examination - July 2016

Subject: Mathematics

Course Unit: MAT112δ (Differential Equations)

Time : One (01) Hour

Answer 02 Questions

1. (a) When V is a function of x , use $y = xV$ to solve the differential equation

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2}.$$

- (b) Solve the differential equation

$$\frac{dy}{dx} = \frac{x \cos x - 2xy}{x^2}$$

by showing that the equation is a perfect differential equation.

- (c) If $y(0) = 3$ then find the solution of the differential equation

$$\frac{dy}{dx} + 2xy = 4x.$$

2. Solve the following differential equations:

(a)

$$\frac{dy}{dx} = \frac{x + 2y + 3}{1 - x - 2y}$$

(b)

$$\frac{dy}{dx} = \frac{x + 2y + 3}{2x + y + 3}$$

3. a) Let $D = \frac{d}{dx}$, and a_1, a_2, a_3 and α be constants.

(i) Show that $(a_1D^2 + a_2D + a_3)\{e^{\alpha x}V(x)\} = e^{\alpha x}(a_1(D + \alpha)^2 + a_2(D + \alpha) + a_3)V(x)$

(ii) If $F(D^2) = \sum_{r=0}^n a_r(D^2)^r$, where $n = 0, 1, 2, 3, \dots$, then show that

$$F(D^2) \sin(\alpha x) = F(-\alpha^2) \sin(\alpha x).$$

b) Solve the differential equation

$$(D^2 + 9)y = x + \sin(2x).$$
