UNIVERSITY OF RUHUNA
FACULTY OF SCIENCE
Bachelor of Science Special Degree in Mathematics
Level II –Semester I Examination, December - 2016

SUBJECT: MATHEMATICS

COURSE UNIT: MSP 4254 – OPERATIONS RESEARCH

INSTRUCTIONS:

- Show all work, simplify your answers and write out your work neatly for full credit.
- Answer **SIX**questions only.
- Time Allowed: **THREE** hours.

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1. A nutritionist is planning a menu in a hospital consisting of two foods A and B. Each ounce of A contains 2 units of fat, 1 unit of carbohydrate and 4 units of protein. Each ounce of B contains 3 units of fat, 3 units of carbohydrates and 3 units of protein. The nutritionist wants the meal to provide at least 18 units of fat, at least 12 units of carbohydrates and at least 24 units of protein. If an ounce of A costs 20 cents and an ounce of B costs 25 cents, how many ounces of each food should be served to minimize the cost of the meal and yet satisfy the nutritionist's requirements?
   (i) Formulate the problem as a linear programming model.
   (ii) Solve the above model graphically.

2. Using the *Revised Simplex Method*, solve the following linear programming problem:

   Maximize $z = x_1 - 3x_3$

   Subject to $x_1 - 2x_2 - x_3 + x_4 = 1$

   $-x_1 + 4x_2 - x_3 \leq 2$

   $-x_1 - 3x_2 + x_3 \leq 3$

   and $x_1, x_2, x_3, x_4 \geq 0.$

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3. Using the *Big M method*, solve the following linear programming problem:

Minimize $z = -3x_1 + x_2 + x_3$

Subject to $x_1 - 2x_2 + x_3 \le 11$

$-4x_1 + x_2 + 2x_3 \ge 3$

$2x_1 - x_3 = -1$

and $x_1, x_2, x_3 \ge 0.$

4. (a) Write down the *Dual* problem of the following Primal problem:

Maximize $z = 6x_1 + x_2 + x_3$

Subject to $4x_1 + 3x_2 - 2x_3 = 1$

$6x_1 - 2x_2 + 9x_3 \ge 9$

$2x_1 + 3x_2 + 8x_3 \le 5$

and $x_1 \ge 0,\ x_2 \le 0,\ x_3$ unrestricted.

(b) Solve the following problem using the *Dual Simplex Algorithm*:

Minimize $z = 2x_1 + 3x_2$

Subject to $3x_1 - 2x_2 \ge 4$

$x_1 + 2x_2 \ge 3$

and $x_1, x_2 \ge 0.$

5. A company has four machines and four jobs to be completed. Each machine must be assigned to complete one job. The times required to setup machines for completing jobs are below:

	Job 1	*Job 2*	*Job 3*	*Job 4*
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

A company wants to minimize the total setup time needed to complete the four jobs.

(i) Formulate the above problem as a linear programming model.

(ii) Solve the above model to find the minimum total setup time.

6. There are three parties who supply the following quantities of coal:

Party 1	14 tons
Party 2	12 tons
Party 3	5 tons
Total	31 tons

There are three consumers who require the coal as follows:

Consumer A	6 tons
Consumer B	10 tons
Consumer C	15 tons

The transportation cost between each party to each customer per ton of coal is as follows:

		Consumer		
		A	***B***	***C***
Party	***1***	6	8	4
	2	4	9	3
	3	1	2	6

Coal should be distributed so that the total transportation cost is minimal. Formulate the above problem as a linear programming model.

Find the schedule of a transportation policy which minimizes the cost.

7. Explain the Exploratory search used in the Hooke-Jeeves pattern search method in step-wise form.

Write down the algorithm of the Hooke-Jeeves pattern search method.

Perform **TWO** iterations of the Hooke-Jeeves pattern search method to solve the following problem:

$$\text{Minimize } (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2.$$

Take the initial point $x^{(0)} = (0, 0)^T$ with $\Delta = (\frac{1}{2}, \frac{1}{2})^T$.

8. (a) Consider the following optimization problem:

Minimize $f(x_1, x_2) = 2x_1^2 + x_2^2 + 4x_1x_2 + 3x_1 + 4x_2 + 7$

Subject to $x \in \Omega = \{x = (x_1, x_2) \in \Re^2 | \ x_1 \geq 1, x_2 \geq 1\}$.

(i) Find the gradient and Hessian of f at the point $(1, 1)^T$.

(ii) Find the directional derivative of f at $(1, 1)^T$ in the direction $d = (d_1, d_2)^T$.

(iii) Does the point $x^* = (1, 1)^T$ satisfy the first-order necessary condition for a minimizer?

(iv) Is the point $x^* = (1, 1)^T$ a minimizer? Justify your answer.

(b) Consider the following minimization problem:

Minimize $(x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$

Subject to $g_1(x) = (x_1 - 5)^2 + x_2^2 - 26 \leq 0$,

$g_2(x) = 4x_1 + x_2 - 20 \leq 0$,

and $x_1, x_2 \geq 0$.

(i) Find whether the points

(a) $(1, 5)^T$ (b) $(0, 0)^T$ (c) $(3, 2)^T$

are feasible.

(ii) Write down the Karush-Kuhn-Tucker (KKT) optimality conditions for the above problem.

(iii) Check whether the point $(3, 2)^T$ is a KKT point.