UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: July 2025

Module Number: IS6303 **Module Name: Mathematical Modelling (C-18)**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1.

a) i.) Briefly explain the stages of 'Mathematical Modelling Process'.

ii.) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to solving each problem? What is the added value in each case?

[3 Marks]

b) A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 1 hour, the number of bacteria increased to 800.

i.) What is the number of bacteria in 5 hours?

ii.) How long will it take to reach 20,000 bacteria?

[3 Marks]

c) A train operator for a particular line is buying some new rolling stock. The trains supplied by one company accelerate in such a way that their speed (V_1), in m/s, varies with time (t) in seconds from a standing start according to the expression:

$$V_1(t) = 35\left[\frac{t^2}{1+t^2}\right].$$

However, the expression for the speed (V_2) of trains from a competitor supplier is:

$$V_2(t) = 100\,[1 - e^{-0.02t}].$$

i.) How far will each train travels in 30 seconds from a standing start?

ii.) Which train would you recommend to the operator and why?

[6 Marks]

Q2.

a) i.) What is meant by the statement "a physical equation is dimensionally homogeneous"?

ii.) Clearly state the 'Buckingham Pi Theorem'.

iii.) The sound power (W), from a turbulent jet through a nozzle is believed to depend on the jet centerline velocity (V), nozzle diameter (D), speed of sound (C) and fluid density (ρ). The temperature and composition of the fluid in the jet is the same as the ambient fluid. Using the Buckingham Pi theorem, find a relation between the sound power and the other parameter(s) of the system.

[5 Marks]

b) An automobile company has two factories. One factory has 400 cars (of a certain model) in stock and the other factory has 300 cars (of the same model) in stock. Two customers have placed orders for this model. The first customer requires 200 cars, while the second customer requires 300 cars. The cost of shipping cars from the factories to the customers is shown in the Table below.

From	To (*Cost in Rs.100s*)	
	Customer 1	**Customer 2**
Factory 1	30	25
Factory 2	36	30

Determine how the company should ship the cars in order to minimize the shipping cost, by using 'Simplex Method'.

[7 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of 'Vogel's Approximation Model (*VAM*)' and 'Modified Distribution Method (*MODI*)' used to solve transportation problems.

[2 Marks]

c) A large manufacturing company is closing three of its existing plants and intends to transfer some of its more skilled employees to three plants that will remain open. The number of employees available for transfer from each closing plant is as follows.

Closing Plant	Transferable Employees
1	60
2	110
3	65
Total	**235**

The following number of employees can be accommodated at the three plants remaining open.

Open Plants	Employees Demanded
A	50
B	75
C	100
Total	**225**

10 employees available for transfer at closing plant 2 confirmed their early retirement and hence not going to employ at any of the open plants.

Each transferred employee will increase product output per day at each plant as shown in the following table. The company wants to transfer employees so as to ensure the maximum increase in product output.

From	To		
	A	B	C
1	6	6	7
2	10	9	12
3	7	10	10

i.) Find the initial feasible solution using *VAM* and calculate its total increase in product output.

ii.) Solve the problem using *MODI*.

iii.)Is the solution obtained in part (ii) optimal? If yes, justify your answer; if not, state why you cannot be certain of its optimality using this method.

[8 Marks]

Q4.

a) Briefly explain what is meant by,

i.) 'Row reductions' and 'Column reductions' performed in assignment problems.

ii.) Prohibited Assignment.

[2 Marks]

b) Give examples (2 each) for maximization and minimization assignment problems occurring in real world with related to engineering/management.

[2 Marks]

c) A health authority needs to distribute emergency medical supplies from 5 Storage Hubs (A, B, C, D, E) to 6 Hospitals (1, 2, 3, 4, 5, 6) that require these supplies urgently. The distances (in kilometers) between each hub and hospital are given in the table below. Find the assignment of medical supplies from supply hubs to hospitals that minimizes the total distance covered during delivery.

Storage Hub \ Hospital	1	2	3	4	5	6
A	12	10	15	22	18	8
B	18	10	25	15	16	12
C	11	10	3	8	5	9
D	14	6	10	13	13	12
E	8	12	11	7	3	10

[6 Marks]

d) Suppose, in above question part (c),

- the transportation is prohibited from Storage Hub C to Hospital 4 due to ongoing road construction works, and
- Hospital 5 already received a supply from a close-by different hospital and hence no longer required a medical supply from Storage Hubs A, B, C, D or E.

What modifications to the transportation tableau would you do to consider these prohibited route and removal of Hospital 5? Briefly explain any specific points of solving this problem with this modification. (Do not perform any calculation again).

[2 Marks]

Q5.

a) i.) Briefly explain what is meant by 'a difference equation'.

ii.) Briefly explain how you can identify the linear stability of a fixed point.

iii.) Briefly explain the difference between monotonic convergence and oscillatory convergence in discrete dynamical systems.

iv.) Find the equilibrium point, if it exists, for the following dynamical system. Is the equilibrium point stable or unstable?

$$u(n) = 0.5u(n-1) + v(n-1) + 3$$
$$v(n) = 0.4u(n-1) - v(n-1) + 6$$

[6 Marks]

b) A manufacturing plant uses a two-stage process to purify wastewater before it is discharged into the environment. The first stage, called the *Sedimentation Tank*, removes most of the suspended solids, while the second stage, the *Biological Treatment Unit*, reduces organic matter. Initially, the sedimentation tank contains 20 units of water (i.e., $S(0)=20$), while the biological treatment unit is empty (i.e., $B(0)=0$).

Each day, 50% of the water in the sedimentation tank is retained for further clarification, 35% is transferred to the biological treatment unit, and the remaining 15% is lost due to sludge removal and evaporation. In the biological treatment unit, 60% of the water is retained for further processing, 10% flows back into the sedimentation tank due to recirculation, and 30% is removed from the system after being treated.

At the end of each day, after these retention, transfer, and loss processes are completed, the plant adds an additional 20 units of new wastewater directly into the sedimentation tank.

Let $S(t)$ represent the amount of water (in units) in the sedimentation tank and $B(t)$ represent the amount in the biological treatment unit, at the end of day t.

i) Write a discrete dynamical system describing the daily change in water content in both treatment stages, $S(t)$ and $B(t)$.

ii) Determine the equilibrium levels of water in the sedimentation tank and the biological treatment unit.

[6 Marks]