University of Ruhuna

Bachelor of Science General Degree
Level I (semester I) Examination – August 2017

Subject: Mathematics
Course Unit: AMT112β (Mathematical Foundation of Computer Science)
Time: Two (02) hours

Answer <u>04 questions</u> only

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(1) (a) Prove that, if x is an positive integer and x^2+4x+1 is an odd number, then x is even using

- (i) direct proof
- (ii) contrapositive proof
- (iii) a proof by contradiction.

(b) Show, using the 1st/2nd principle of Mathematical Induction, that

(i) for all $n \geq 5$, $4n < 2^n$.

(ii) $a_n < \left(\frac{7}{4}\right)^n$, where a_n s are the Lucas numbers defined by

$$a_n = \begin{cases} 1, & \text{if } n = 1 \\ 3, & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & if\ n > 2. \end{cases}$$

(c) State the Generalized pigeonhole theorem.

(i) There are 54 students in a class. Show that there are at least three students whose names start with the same letter in the English alphabet.

(ii) What is the least number of students that must be in a class to ensure that 20 students get the same grade one of A, B, C or D?

(2) (a) Consider the following argument:

If you come to meet me I will explain the problem to you. If you do not come to meet me then I will go to the library. Therefore I will explain the problem to you or I will go to the library.

Symbolize and test the above argument for the validity by using

- (i) a truth table
- (ii) pattern proof

(b) Consider the following sentences:

> All players are clever.
> Anyone who is clever and dedicated can play the game well.
> Anyone who is playing the game well will win his/her game.
> Dakshawathi is a dedicated player.

(i) Represent the above facts in axiom forms.
(ii) Translate the above axioms into clausal form.
(iii) Use these clausal forms to prove that " Dakshawathi will win the game."

(3) (a) Explain what is meant by a tautology and a contradiction.
Using a truth table, determine whether the following is a tautology, contradiction or neither:

$$[(A \wedge B) \rightarrow C) \Leftrightarrow (A \rightarrow (B \rightarrow C)).$$

(b) A computer has a word length of 8 bits and uses two's complement method for calculations. Translate -101 into the number format used by the above computer.

(c) Explain the method of simplifying 63 – 71 using two's complements in an 8-bit word-length computer.

(d) Explain the method of performing the calculation $(101_{10})/(13_{10})$ in a computer with 8-bit word-length that uses two's complement.

(4) (a) Consider the logic formulas G_1 and G_2:

$$G_1: (\forall X)\,[P(X) \vee Q(X)] \rightarrow (\forall X)P(X) \vee (\forall X)Q(X);$$
$$G_2: (\exists X)P(X) \rightarrow (\forall X)P(X).$$

Obtain the intuitive meanings of G_1 and G_2 under the interpretation I over the set of integers; under which
P(x) means that x is even and
Q(x) means that x is odd.

(b) Define the dual of a proposition considering a Boolean Algebra $\mathfrak{B}$.
Find the dual of the Boolean expression $\overline{(A.\overline{B})} + (A.\overline{B}) = 1$.

For all A, B, C in $\mathfrak{B}(.,+,\bar{\ },0,1)$, prove that

(i) $A.B = \overline{(\overline{A} + \overline{B})}$,

(ii) $\overline{A+B} = \overline{A}.\overline{B}$.

(c) Draw a state diagram for a finite state machine which will accept set of all strings that ends in 010; where input symbols consists of {0,1}.

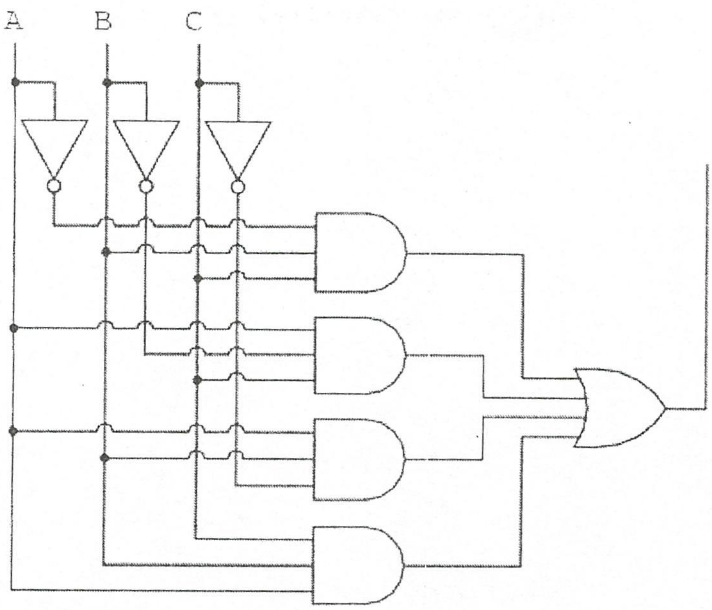
(5) (i) Define what is meant by

(α) a sum-of-products expression

(β) a complete-sum-of-products expression

(γ) a minimal sum-of-products expression.

(ii) Write a Sum Of Products expression for the following logic circuit.



(iii)Using the laws in Boolean algebra, show that the minimal sum-of-products expression equivalent to this is $A.B + B.C + A.C$.

(iv) Verify yor answer in part (iii) using a Karnaugh map.

(iv) Sketch the logic circuit for the minimal sum-of-products expression you obtained in (iii) above using AND, OR and NOT gates.

(6). (i) Use the expand, guess and verify method to show that the closed-form-solution of the recurrence relation

$$S(n) = 8 * S(n-1) \quad \text{for } n \geq 2$$

with the base value S(1) = 4 is given by

$$S(n) = 2^{3n-1}.$$

(ii) Show that the solution to the linear first-order recurrence relation of the form

$$S(n) = S(n-1) + g(n)$$

with constant coefficients and the base value S(1) is given by

$$S(n) = S(1) + \sum_{i=2}^{n} g(i).$$

The first four terms of a sequence are given by 5, 12, 21 and 32.

Find a recurrence formula of the form $S(n) = S(n-1) + g(n)$ for the n^{th} term in the sequence.
Hence find a formula for the n^{th} term in the sequence.

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