University of Ruhuna
Bachelor of Science General Degree
(Level II) Semester I Examination - August/September 2017

Subject : Applied Mathematics
Course unit: AMT212β/ MAM2123 (Computational Mathematics)

Time :Two (02) Hours

Answer 04 Questions only.
Allowed to use calculators only supplied by the University.

---

1. a) Explain the following terms.
   (i) Absolute error.
   (ii) Relative error.

   b) Let x_a and y_a be approximate values of two numbers whose true values are x_t and y_t and, the corresponding absolute errors are e_x and e_y respectively.
   Show that $|(x_t + y_t) - (x_a + y_a)| \leq e_x + e_y$.

   c) Explain the form of a floating point number in the finite number system.
   Using IEEE single precision format show that
   (i) $147.625_{10} = 4313A000_{16}$
   (ii) $\mathrm{BF800000}_{16} = -1_{10}$

   d) Assuming the power function model of the form $y = ax^b$ for the data set (x_i, y_i); $i = 1, 2, 3, 4, 5$ given below, apply the least square approximation to find a and b.

x_i	1	2	3	4	5
y_i	0.5	2	4.5	2	12.5

   (In the usual notation, for the least square line $y = a + bx$ the least square estimates are given by $\hat{b} = \dfrac{\sum_{i=1}^{n} x_i y_i - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)/n}{\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2/n}$ and $\hat{a} = \bar{y} - \hat{b}\bar{x}$)

2. a) Write down the algorithm for. Newton-Raphson method for finding an approximate root of non-linear equation $f(x) = 0$.
Show that the convergence of Newton-Raphson method is of order two.

To find the approximate root of the equation $x - e^{-x} = 0$, use Newton-Raphson method with the initial point $x_0 = 1$ up to four iterations and tabulate the function value and solution at each iteration.

b) Constructing fixed point iteration formula, perform four iterations with initial point $x_0 = 1$ to obtain approximate solution for the equation $x^2 - 5 = 0$ using each of the following re-arrangements.

(i) $x = 5/x$

(ii) $x = \frac{(5/x)+x}{2}$

Describe the convergence property of two re-arrangements.

---

3. a) Obtain the second degree Taylor polynomial approximation for $f(x) = (1+x)^{1/2}$ about $x = 0$; where $x \in [0, 1]$.
Approximate $f(0.03)$ using this polynomial approximation and find the actual error.
Find the error bound for $x \in [0, 1]$.

b) Consider the data set with x values 0, 1, 2, 3 and 4, and the corresponding function values 0, 7, 26, 63 and 124 respectively.
Determine Newton's divided difference interpolation polynomial using all the data.
Find the function value at $x = 1.5$.

---

4. Write down the boundary conditions for natural cubic spline for (n+1) points x_0, x_1, x_2,...., x_n whose function values are $f(x_0)$, $f(x_1)$, $f(x_2)$,...., $f(x_n)$ respectively. in the usual notation.
Find the natural cubic spline that passes through the points (1, 1), (2, 2), (3, 5) and (4, 11).
Compute the cubic spline function value when $x = 1.5$.

---

5. a) Using the Taylor series expansion of a continuously differentiable function f at the point $x + h$ for some $h > 0$, obtain the forward difference formula for $f'(x)$ to first order approximation ; where $f'(x)$ is the first derivative of the function f at x.

Let $f(x) = sinx$
Using the above formula, find the approximate value of $f'(x)$ at $x = 0.45$ (radians)
Take $h = 0.01$.
Also obtain the error bound of the approximation.

Continued.

b) In the usual notation, derive three point formula to find the first derivative of a function f, at points which are equally spaced.
A car moves in a horizontal way. The distance it moved (in *meters*) at 5 different times (in *seconds*) are recorded as below:

Time (t)	5	6	7	8	9
Distance(S)	10	14.5	19.5	25.5	32

Use the most appropriate three point formula and approximate the speed (in ms^{-1}) of the car at $t = 5$, $t = 7$ and $t = 9$.

---

6. a) In the usual notation, write down Trapezoidal rule used in approximating $\int_a^b f(x)dx$.

Using Trapezoidal rule, evaluate the value of $\int_1^2 (x^3 + 1)dx$.

b) Using the Lagrange interpolation polynomial for $n = 2$, obtain Simpson's $\frac{1}{3}$ rule, $\int_a^b f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$ with the usual notation.
; where $h = \frac{b-a}{2}$, $x_0 = a$, $x_1 = \frac{a+b}{2}$ and $x_2 = b$

Evaluate $\int_0^{\pi/2} \sqrt{(sinx)}dx$ using Simpson's $\frac{1}{3}$ rule.

c) Obtain the expression of the composite Simpson's rule to find $\int_a^b f(x)dx$ for odd number of points $a = x_0, x_1, x_2, ..., x_{2N} = b$.

Use composite Simpson's rule and evaluate the integral $\int_0^{\pi/2} \sqrt{(sinx)}dx$, when 5 points are involved.
