University of Ruhuna

Bachelor of Science General Degree

Level I (Semester II) Examination - 2018

Subject: Industrial/ Applied Mathematics
Course Unit: AMT121β/ IMT121β
(Classical Mechanics-II- Statics) .

Time: Two (02) Hours

Answer All Questions

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1. (a) State the Lami's theorem for three forces.

   (b) Let PQR be any triangle. The forces X, Y and Z act along the sides QR, RP and PQ of the triangle PQR, respectively.

   (i) Find the moment of the three forces about the incentre of the triangle PQR.

   (ii) Find the moment of the three forces about the circum centre of the triangle PQR.

   (iii) If the resultant of the three forces passes through both incentre and circumcentre of the triangle PQR, show that

$$\frac{X}{\cos Q - \cos R} = \frac{Y}{\cos R - \cos P} = \frac{Z}{\cos P - \cos Q}$$

   (c) Assuming that the three forces P, Q, R shown in the following figure are in equilibrium, show that

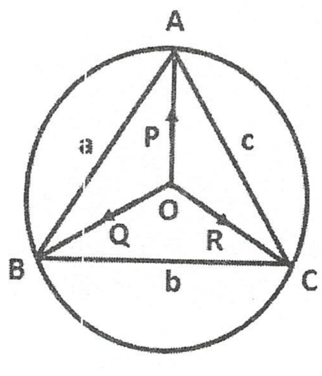
   (i)

$$\frac{P}{\sin 2A} = \frac{Q}{\sin 2B} = \frac{R}{\sin 2C},$$

   (ii)

$$\frac{P}{b^2(a^2+c^2-b^2)} = \frac{Q}{c^2(a^2+b^2-c^2)} = \frac{R}{a^2(b^2+c^2-a^2)},$$

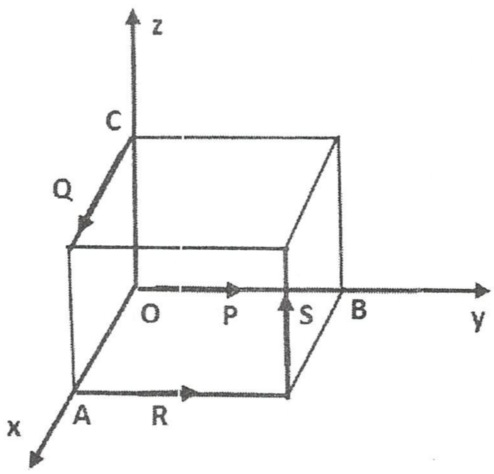
   where O is the circumcentre of the triabgle ABC.



2. (a) A system of forces is equivalent to a single force $\underline{R}(\neq \underline{0})$ and a couple $\underline{G}(\neq \underline{0})$ at the origin O. Let X, Y, Z and L, M, N be the components of $\underline{R}$ and $\underline{G}$ respectively. Also assuming that the system can be reduced to a wrench at another point O', show that the equation of the central axis of the wrench is given by

$$\frac{L - yZ + zY}{X} = \frac{M - zX + xZ}{Y} = \frac{N - xY + yX}{Z},$$

(b) Forces P, Q, R and S are acting along the edges of a cube of length a as shown in the following figure where O is the origin and OA, OB, OC are the coordinate axes.



By using the above diagram,

(i) find the equation of line of action of each force,

(ii) find the components of the forces paralleled to the coordinate axes,

(iii) find the components of the moment about origin of the force system,

(iv) find the equation of the central axis.

3. (a) In the usual notation, obtain the relation

$$EI\frac{d^4y}{dx^4} = w(x),$$

assuming that

$$EI\frac{d^2y}{dx^2} = M$$

for a uniform beam, where w is the weight per unit length, E is the modulus of elasticity and I is the moment of inertia of the cross section about its neutral axis.

Continued...

(b) A uniform beam AB of length $2l$ and weight w per unit length is fixed horizontally to a wall from end A and the end B is freely hung.

(i) Express clearly the boundary conditions you may use.

(ii) Find the equation of the elastic curve.

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4. (a) Show that

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}),$$

you may use the relation

$$\sinh x = \frac{1}{2}(e^x - e^{-x}).$$

Hence, show that

$$\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x.$$

(b) In the usual notation, obtain the following relations

(i) $s = c\tan\psi$,

(ii) $x = \ln|\sec\psi + \tan\psi|$,

(iii) $y^2 = c^2 + s^2$,

for a uniform catenary.

(c) A uniform closed string is hung over a smooth circular laminar of radius r in the vertical plane. The length of the closed string is greater than the circumference of the circle. Further, 3/4 of the circumference of the circular laminar contact with the closed string. Show that the length of the closed string is given by

$$r\left\{\frac{3\pi}{2} + \frac{\sqrt{2}}{\log(1+\sqrt{2})}\right\}.$$
