University of Ruhuna

Bachelor of Science General/Special Degree
Level II (Semester II) Examination - January 2018

Subject: Industrial Mathematics
Course Unit: IMT2b2β/MSP3b9β (Mathematical Computing(Repeat Examination))
Time : Two (02) Hours

Answer All questions.

Student Number: .....

Read the following Instructions carefully.

(a) No calculators are required.

(b) Your home directory at the exam shall NOT be your usual home directory.

(c) Create a folder with your index number and save all files inside it.

(d) Save each answer while doing the paper.

(e) Save each question with the question number inside the created folder.(Eg: Q1.wxm)

(f) At the end of the examination you MUST submit this paper along with any extra sheets you may use, attached to this paper.

1. Use WXMaxima to calculate the numerical value of the followings and write down the answers.

(i) $\frac{1}{5} + \frac{5}{11} - 5$ .....

(ii) $e^{-4} + \cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{8}\right)$ .....

(iii) $\sqrt{5} + e^{\pi} + \frac{40}{8}$ .....

(iv) $\sin 35^o + (0.7734)^{-0.456} + e^{4.3}$ .....

(v) $5^{\ln 34} + e^{0.234} + 3^{\log_{10} 34}$ .....

(vi) $\log_{2.456} 12 + \log_{0.123} 10 + \ln 23.1543$ .....

(vii) $\frac{1}{3}\left(\frac{\ln 15}{\log_{1.52} 4}\right)$ .....

(viii) $4e^{1.53}\log_{3.5} 15$ .....

(ix) $e^{1.23 \ln 4}$ .....

(x) $\log_{\pi} e$ .....

Continued.

2. (a) Write down expressions in WXMaxima for the followings:

(i) $\sum_{i=1}^{p} ({x_i}^2 + x_i + 2)$

(ii) $\sum_{t=1}^{k} (2^t + t^2)$

(iii) $\prod_{p=1}^{\infty} p^x$

(iv) $\prod_{k=1}^{\infty} a^{T_k}$

(b) Answer the following questions using WXMaxima.

(i) Solve $x^5 + 6x^4 - 13x^3 - 90x^2 - 72x$ for x.

(ii) Find numerical value of $\frac{d}{dx}(\ln x^2 - 2x + 5)$ at x=2.

(iii) Find $\frac{\partial^3}{\partial y \partial x^2}(-y\sin(2x) + 5x^2 e^{-3y})$

(iv) Find the jacobian matrix of the function

$$\underline{f}(x, y) = (x^2 - 2xy + 2y + x - 1, e^{x+y^2})$$

(v) Evaluate $\int_0^{\infty} 5e^{-5t}\, dt$

(vi) Integrate $\int_0^1 \int_a^b \frac{y}{b-a}\, dxdy$

(vii) Find partial fraction of $\frac{4x-1}{(x^2 - 4x + 4)}$

(viii) Compute the limit of

$$\lim_{n\to\infty} \frac{1}{\left(1 + \frac{1}{n}\right)^n}$$

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3. Let Y be a poissonly distributed random variable with event rate λ. The probability of observing k events in an interval is given by the equation

$$Pr(Y = k) = \frac{e^{-\lambda}\lambda^k}{k!} \qquad k = 0, 1, 2, ...$$

a) Define above function in WXMAXIMA.

b) Suppose web site of the Department of Mathematics is visited averagely by 4 people daily. Find the following pobabilities.

(i) Exactly 2 people will visit the web site.

(ii) At least 3 people will visit the web site.

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4. The plant manager of a company that manufactures office equipment's is attempting to determine the pooled variance between two assemble methods of a new chair. n_1=15 random selected workers each assembled the chair using method A and another n_2=15 workers using method B. The assembly times in minutes fo each worker were recorded as shown in the table below.

Assemble times	6.3	5	6.1	6.1	6.4	5	5.9	6.2
by method A	6.1	6.1	6.9	5.2	7.1	5	6.6	
Assemble times	5.2	6.5	7.2	5.9	5.9	6.7	5.9	6.5
by method B	6.1	4.9	5.7	6.7	5.3	5.8	5.7	

Find the pooled variance($S_p{}^2$) between two assemble methods. Note that $S_p{}^2$ is given by the formula

$$S_p{}^2 = \frac{(n_1 - 1)S_1{}^2 + (n_2 - 1)S_2{}^2}{n_1 + n_2 - 2};$$

where $S_1{}^2$ and $S_2{}^2$ are the sample variances of method A and B respectively and the sample variance and mean are given by following expressions.

$$S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \quad \text{and} \quad \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}.$$

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5. (a) (i) Create a 4 ×4 identity matrix.

(ii) Create a diagonal matrix of size 6 with diagonal element $\sin(x)$.

(b) Let

$$\text{A}=\begin{bmatrix} 4 & 4 & -3 \\ 2 & 6 & 3 \\ -4 & 2 & 1 \end{bmatrix} \text{ and B}=\begin{bmatrix} 6 & 2 & 2 \\ 1 & 9 & 1 \\ -3 & 2 & 8 \end{bmatrix}$$

(i) Input the matrix A and B in Maxima.

(ii) Find A + B.

(iii) Find 3A.

(iv) Find B^2

(iv) Find A.B

(vi) Find inverse of A + B.

(vii) Find transpose of A + B.

(viii) Find the determinant of A + B.

(ix) Obtain characteristic equation of A.

(x) Divide the first row of matrix A by 2.

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6. (a) Plot the functions $f(x) = \frac{1}{2\pi}e^{-\frac{x^2}{2}}$ and $g(x) = \frac{1}{\pi(1+x^2)}$ on the same graph in the range of -3 ≤ x ≤ 3. Name the graph f(x) as Gaussian distribution and the graph g(x) as Cauchy distribution. The vertical axis should be labeled as density.

(b) The Binormal distribution is given by $f(x,y) = \frac{1}{2\pi}\exp(-\frac{1}{2}(x^2+y^2))$.

(i) Plot the function f(x,y),

Continued.

(ii) Obtain contour plot, in the range -3 $\leq$ x $\leq$ 3 and -3 $\leq$ y $\leq$ 3

(c) Plot the graph represented by the parametric equations $x = t^5 + \sin(2\pi t)$ and $y = t + e^t$ for -2.5 $\leq$ t $\leq$ 2.5. The horizontal and vertical axis should be labeled as x(t) and y(t) respectively.

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7. (a) (i) Define the following function in Maxima

$$f_(x) = \begin{cases} (x+5)^2 & ;x< -5 \\ (x+5)/2 & ; -5 \leq x<0 \\ (-x+5)/2 & ;0 \leq x<5 \\ (x-5)^2 & ;x \geq 5 \end{cases}$$

(ii) Plot the above function in the interval [-10,10].

(b) (i) Given a quadratic equation $ax^2 + bx + c = 0$. If $b^2 - 4ac$ is non-negative, the roots of the equation can be solved with the following formulae,

$$root1 = \frac{1}{2a}(-b + \sqrt{b^2 - 4ac})$$

$$root2 = \frac{1}{2a}(-b - \sqrt{b^2 - 4ac})$$

Write a Maxima program inside a block to read in coefficients a,b and c, and compute and display the roots. If the discriminant $b^2 - 4ac$ is negative the equation has complex roots. Thus, this program should solve the equation if the discriminant is non-negative and show a massage otherwise.

(ii) Using the above program solve following quadratic equations.

(i) $x^2 - 5x + 6 = 0$

(ii) $x^2 - 4x + 4 = 0$

(iii) $x^2 + 2x + 5 = 0$

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8. a) Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

b) A container in the shape of a right circular cylinder with no top has surface area $3\pi\ ft^2$.Giving comment for every step answer the following,

(i) What height h and base radius r will maximize the volume of the cylinder?

(ii) What is the maximum volume of the cylinder?
