University of Ruhuna

B.Sc.(General) Degree

Level II (Semester II) Examination - January- 2018

Subject: Mathematics
Course Unit: MAT224δ(Geometry)   Time: One (01) Hour

Answer **Two (02) Questions only.**

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1. (a) Let P and Q be the two points with coordinates $(-3, 1, 1)$ and $(3, 4, 2)$ respectively. A point R lies on PQ such that $PR : PQ = 1 : 3$. Find the equation of the plane through R perpendicular to PQ.

(b) Consider a triangle with sides of lengths $2a, 2b, 2c$. The middle points of sides are on the axes. Show that the equation of the plane through middle points is

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1,$$

where

$$\alpha^2 = \frac{b^2 + c^2 - a^2}{2}, \beta^2 = \frac{a^2 + c^2 - b^2}{2}, \text{ and } \gamma^2 = \frac{a^2 + b^2 - c^2}{2}$$

(c) Two straight lines are given by,

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2} \text{ and } \frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}.$$

Find

(i) the shortest distance between the two lines.

(ii) the equation of the line with the shortest distance.

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2. (a) Show that the necessary condition for the plane $lx + my + nz = p$ to touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ is

$$(lu + mv + nw + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$$

(b) Find the equations of two tangent planes to the sphere $x^2 + y^2 + z^2 + 2x = 15$ which passes through the line $x + y = 7, x - 2z = 3$.

(c) (i) Show that the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at the point (x_1, y_1, z_1) is

$$xx_1 + yy_1 + zz_1 + (x + x_1)u + (y + y_1)v + (z + z_1)w + d = 0$$

(ii) Find the equation of the sphere which touch the sphere $4(x^2+y^2+z^2)+10x-25y-2z=0$ at the point $(1,2,-2)$ and passes through the point $(-1,0,0)$.

(d) Obtain the equation of the sphere having the circle $x^2+y^2+z^2+6y+6z+10=0, x+y+z=5$ as the great circle. Hence, find its center.

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3. (a) (i) Obtain the equation of the tangent plane and the normal line to the conicoid $Ax^2+By^2+Cz^2=1$ at the point (α,β,γ) on it.

(ii) Find the equations of the tangent planes to the surface $4x^2-5y^2+7z^2+13=0$, parallel to the plane $4x+20y-21z=0$. Find their points of contact.

(b) If a tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the co-ordinate axes at the points A, B and C, prove that the locus of centroid of the triangle ABC is $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9$.

(c) Obtain the two systems of generators of the hyperboloid of one sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1.$$

Show that

(i) no generators of the same system intersect, and

(ii) any two generators of the different systems intersect.

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