UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 4 Examination in Engineering: February 2020

Module Number: IS4301

Module Name: Probability and Statistics (New Curriculum)

[Three Hours]

[Answer **all questions**, each question carries fourteen marks]

Q1. a) The scores of a sample of 20 students on college entrance examination are

36 44 78 84 66 48 50 69 74 70 52 54 59 61 57 56 60 58 64 65

i Construct a relative frequency histogram of the data (Consider equal class width and use 35-45 as the beginning class interval).

ii If the college wants to accept the top 35% of the applicants, what should the minimum score be?

iii If the university sets the minimum score at 45, what percent of the applicants will be accepted?

[3.0 Marks]

b) Table 1.1 shows the descriptive statistics for the times (in seconds) of the 28 swimmers in the final round of the male students' 100-meter backstroke event in the school aquatic competition.

Table 1.1

Mean	Median	St Dev	Minimum	Maximum	Q1	Q3
33.36	34.5	8.73	11	46	28.25	39.75

i Calculate $1.5(IQR)$ and subtract it from Q_1 and add it to Q_3 to determine outliers.

ii From the descriptive statistics, construct a box-and-whisker plot and comment on the shape of the distribution.

iii Based on the shape, give measure of the center of the distribution.

[7.0 Marks]

c) A new analytical method is used to detect three different contaminants: organic pollutants, volatile solvents, and chlorinated compounds. The makers of the test claim that it can detect high levels of organic pollutants with 99.7% accuracy, volatile solvents with 99.95% accuracy, and chlorinated compounds with 89.7% accuracy. If a pollutant is not present, the test does not signal. Samples are prepared for the calibration of the test and 60% of them are contaminated with organic pollutants, 27% with volatile solvents, and 13% with traces of chlorinated compounds. A test sample is selected randomly.

i What is the probability that the test will signal?

ii If the test signals, what is the probability that chlorinated compounds are present?

[4.0 Marks]

Q2. a) In a quality control program, a manufacturer randomly selects two glass sheets from each lot of seven for inspection.

i List the different possible outcomes.

ii If the first, second and fourth glass sheets are the only defectives in a lot of seven, find the probability distribution of the number of defective glass sheets observed among those inspected.

iii Find the cumulative distribution function $F(x) = P(X \leq x)$ for all x and use it to calculate $P(2 \leq X \leq 5)$.

iv Find the mean and variance of the defective glass sheets.

[8 Marks]

b) The moment-generating function of the random variable X is given by,

$$M_X(t) = E(e^{tX}).$$

Then

$$\left.\frac{d^r M_X(t)}{dt^r}\right|_{t=0} = \mu_r' \text{ , where } \mu_r' = E(X^r) \text{ , } r = 1,2,3,....$$

i Find the moment-generating function of the random variable X having a Poisson distribution.

ii Hence, find the mean and variances of the random variable X.

[6.0 Marks]

Q3. a) Let $x_1, x_2, ..., x_n$ are a random sample from a normal distribution with parameters μ and σ^2. The probability density function is given by

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, & -\infty < x < \infty \\ 0, & otherwise \end{cases}$$

i Find the maximum likelihood estimators of μ and σ^2.

iii Use the following data to find the estimate for μ and σ^2.
2, 5, 7, 43, 18, 19, 16, 11, 22, 4, 34, 19, 21, 23, 6, 21, 7, 12

[7 Marks]

b) A study was carried to test the thickness of plastic sheets produced by a machine as the viscosity of the liquid mold makes some variation in thickness measurements. Thickness measurements (in millimeters) of ten plastic sheets produced on a particular shift are 226, 228, 226, 225, 232, 228, 227, 229, 225, 230.
It is stated that the true standard deviation of thickness exceeds 1.5 millimeters, there is cause to be concerned about the product quality.

i State the assumption can be made about the population distribution.

ii Do the data substantiate the suspicion that the process variability exceeded the stated level on this particular shift?

iii Construct a 95% confidence interval for the true standard deviation of the thickness of sheets produced on this shift.

iv If the population variance is 6, then Construct a 95% confidence interval for the true mean of the thickness of sheets produced on this shift.

[7.0 Marks]

Q4. a) A student union of a college A claims that the mean student's food expenses is 450 rupees a day because there is no canteen in the college premises. To verify this claim, a study of 20 randomly selected students was investigated and collected the daily expenses for foods. The data as follows:

440, 456, 432, 462, 418, 444, 486, 402, 422, 460,
462, 454, 440, 472, 438, 475, 464, 455, 424, 470

Do the data contradict the students' union claim?

[5.0 Marks]

b) A study was carried to test the relationship between facility conditions at gasoline stations and aggressiveness in the pricing of gasoline. The corresponding observed and expected counts (given in brackets) based on a sample of 400 stations are given in Table 4.1.

Table 4.1

		Observed Pricing Policy		
		Aggressive	Neutral	Nonaggressive
Condition	Substandard	25 (21.56)	35 (30.94)	15 (22.5)
	Standard	40 (50.31)	60 (72.19)	75 (52.5)
	Modern	50 (43.13)	70 (61.88)	30 (45.0)

Does the data suggest that the facility conditions and pricing policy are independent of one another? Use a chi-square test at 0.05 level of significance.

[4.0 Marks]

c) A study was designed to investigate the iron content of some of the foods cooked in Aluminum, Clay and Iron pots. The iron content (mg/100g food) of the food cooked in each of the three types of pots is measured and the relevant data are summarized in Table 4.2.

Table 4.2

Type of Pot (i)	n_i (i^{th} sample size)	$y_{i.}$	y_{ij}^2
Aluminum	4	8.24	17.16
Clay	4	8.72	20.16
Iron	4	18.72	88.8

Use this data and a level of 0.01 to test the null hypothesis of no difference in mean iron content of foods for three types of pots.

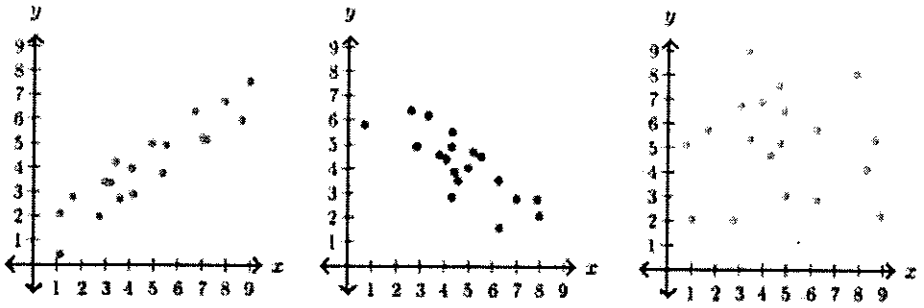
Total sum of squares: $SST = \sum_{i=1}^{I}\sum_{j=1}^{J} y_{ij}^2 - \frac{y_{..}^2}{IJ}$

Treatment sum of squares: $SSTr = \frac{1}{J}\sum_{i=1}^{I} y_{i.}^2 - \frac{y_{..}^2}{IJ}$

$$SST = SSTr + SSE$$

[5.0 Marks]

Q5. a) Describe the relationships between the two variables x and y given by the scatter diagrams in Figure 5.1.



i) ii) iii)

Figure 5.1: Scatter Diagrams

[1.0 Marks]

b) Table 5.1 shows the data on x – normal stress on a certain type of metal test specimen and y = shear resistance.

Table 5.1

x	26.8	25.4	28.9	23.6	27.7	23.9	24.7	28.1	26.9	27.4
y	26.5	27.3	24.2	27.1	23.6	25.9	26.3	22.5	21.7	21.4

i The least square estimates of the slope β_1 and the intercept β_0 of the true regression line respectively are:

$$\hat{\beta}_1 = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2} \text{ and } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

If the simple linear regression model is suitable for the data, find the regression equation.

ii Estimate the shear resistance for a normal stress of 24.5 kilograms per square centimeter.

[8.0 Marks]

c) Table 5.2 represents the data on wear of a bearing y and its relationship to oil viscosity x_1 and load x_2.

Table 5.2

y	x_1	x_2
193	1.6	851
172	22	1058
113	33	1357
230	15.5	816
91	43	1201
125	40	1115

i If the linear regression model is suitable for the data, state the regression equation in matrix notation.

ii Use the data in Table 5.2 to represent the matrices for $\boldsymbol{Y}, \boldsymbol{X}, \boldsymbol{\beta}$ and $\boldsymbol{\varepsilon}$.

iii If $\underline{\hat{\beta}} = (350.9943 \quad -1.272 \quad -0.1539)'$, write down the estimated regression equation.

iv Hence, predict wear when oil viscosity is 20 and load is 1200.

[5.0 Marks]