



UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: February 2020

Module Number: EE6302

Module Name: Control System Design

[Three Hours]

[Answer all questions, each question carries 12 marks]

Note: A table of Laplace transforms is attached.

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- Q1 a) i) Using a block diagram show the components of a closed-loop control system.
- ii) Briefly explain the purpose of the controller, the actuator and the sensor in a closed-loop control system?
- iii) What is meant by a disturbance to a control system?
- [4 Marks]
- b) Consider the system shown in Figure Q1(b).
- i) Determine the value of k so that the damping ratio of the system is 0.5.
- ii) In the s -plane, show the poles of the system indicating the damping ratio and the undamped natural frequency of the system.
- iii) Calculate the time function of the unit-step response of the system when the damping ratio of the system is 0.5.
- [4.5 Marks]
- c) Consider the system shown in Figure Q1(c). $W(s)$ is the disturbance to the system.
- i) Determine whether the system can track a unit step reference in the absence of the disturbance.
- ii) Can the system reject a unit step disturbance? Explain your answer using suitable calculations.
- [3.5 Marks]
- Q2 a) i) In terms of the characteristic equation of a system, what is the necessary condition to be fulfilled in order to have a stable system?
- ii) State the Routh's necessary and sufficient condition to have a stable system.
- [1.5 Marks]
- b) The characteristic equation of a system is given by
- $$s^4 + 2s^3 + (4 + k)s^2 + 9s + 25 = 0.$$
- Using Routh's stability criterion, determine the range of k so that the system to become stable.
- [2.5 Marks]

- c) Explain a method to check the stability, when the transfer function of the system is not known.

[1.5 Marks]

- d) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name the matrices in your matrix equations.
ii) Consider the RLC circuit shown in Figure Q2. The input voltage is V_i and the output voltage is V_o , the voltage across the capacitor. Writing differential equations, represent the system in state-variable form. Take the state vector x as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where } x_1 = V_o \text{ and } x_2 = \dot{V}_o.$$

Input and output of the system is V_i and V_o , respectively.

- iii) Using the state-variable form you obtained in part ii), derive the transfer function of the system.

[6.5 Marks]

- Q3 a) i) State three advantages of the closed-loop control when it is compared to the open-loop control.
ii) Consider the closed-loop control system shown in Figure Q3 where

$$G(s) = \frac{1}{(s + \tau_1)(s + \tau_2)}$$

τ_1, τ_2 can be real or complex numbers.

The controller used here is a proportional controller with gain K . Explain how this closed loop control system can improve the transient response when compared with its open loop control system.

- iii) State the definition of the root locus.

[5.0 Marks]

- b) Consider the closed loop control system shown in Figure Q3 where

$$G(s) = \frac{(s^2 - 2s + 5)}{(s + 2)(s^2 + 3s + 8)}$$

The reference r in this system is a unit step.

- i) Plot the root locus of this system.
ii) Calculate the imaginary axis crossings of the root locus.
iii) Find the range of gain K where the closed-loop system is stable.
iv) Find the steady state error of the closed-loop system's output y for gain $K = 2$.
v) Design a lag compensator to improve the steady state error of the system in part b)iv) by a factor of 10.

[7.0 Marks]

- Q4 a) i) Consider the closed-loop control system shown in Figure Q4. The reference, r in this system is a unit step. What types of compensators can be used for $G_c(s)$ for the following purposes?
- I. Improve the steady state error of the output y .
 - II. Reduce the percent overshoot of the output y .
 - III. Decrease the settling time of the output y .
 - IV. Improve the steady state error and decrease the settling time of the output simultaneously.
- ii) Consider the closed-loop control system shown in Figure Q4. The reference r in the system is a unit step. Prove that the steady state error of this system can be driven to zero when Proportional plus Integral (PI) compensator is used. Assume that $G(s)$ does not contain a zero at the origin.

[4.0 Marks]

- b) Consider the closed-loop control system shown in Figure Q4 where

$$G(s) = \frac{K}{(s + 10)(s^2 + 4s + 2)}$$

The reference r in this system is a unit step. You need to design the compensator $G_c(s)$ for this system to improve the settling time of the output y with the use of the root locus technique.

- i) Calculate the gain K required to yield 20% overshoot of the output y .
- ii) State the assumptions you made in calculating the gain K in part b) i). Justify your assumptions.
- iii) Estimate the settling time and the peak time of the system response for the gain you found in part b) i).
- iv) Design a suitable compensator to improve the settling time of the system response by a factor 3.

[8.0 Marks]

- Q5 a) i) Draw the asymptotic approximations of the bode plots for the system with the transfer function

$$G(s) = \frac{12(s + 10)}{(s + 2)(s + 3)}$$

- ii) Explain how you determine whether an open loop system is stable in closed loop, with the use of frequency response of the open loop system.

[4.0 Marks]

- b) Consider the closed-loop control system shown in Figure Q4 where

$$G(s) = \frac{36.7}{(s + 3)(s^2 + 4s + 5)}$$

The reference r in this system is a unit step. The percent overshoot of the system response is 20%. You need to design the compensator $G_c(s)$ for this system to improve the steady state error of the output y with the use of the frequency response technique. In order to answer the following questions you may require following formulas, where all the notations have usual meaning.

$$\phi_M = \tan^{-1} \left(\frac{2\xi}{\sqrt{-2\xi^2 + \sqrt{1+4\xi^4}}} \right) \text{ and } \omega_{BW} = \frac{4}{T_s\xi} \sqrt{(1-2\xi^2) + \sqrt{4\xi^4 - 4\xi^2 + 2}}$$

- i) Calculate the steady state error of the output y without the compensator $G_c(s)$.
- ii) Design a lag compensator for the system above to improve the steady state error by a factor of 5 without significantly changing the percent overshoot.

[8.0 Marks]

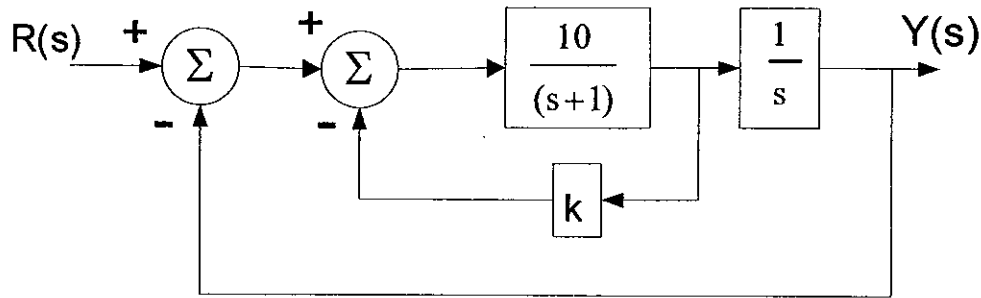


Figure Q1(b)

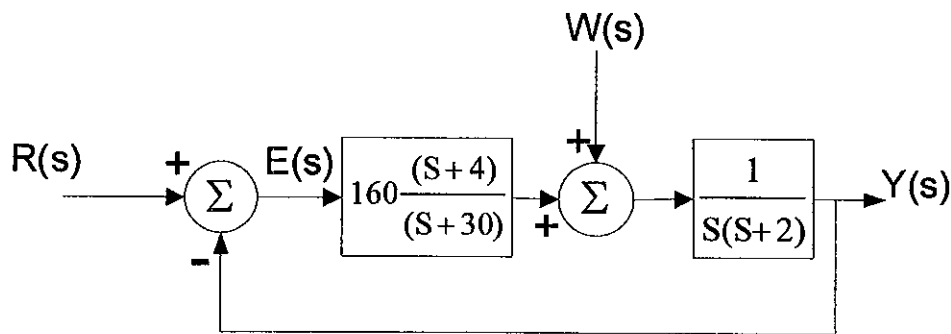


Figure Q1(c)

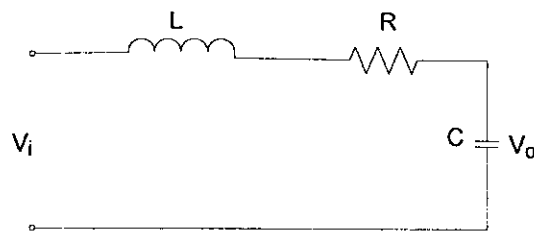


Figure Q2

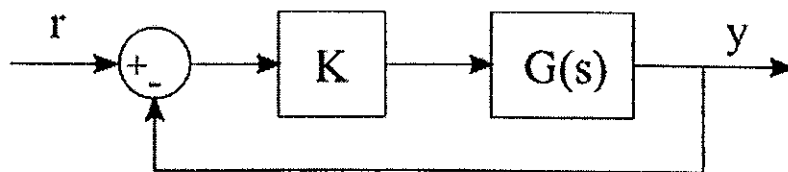


Figure Q3

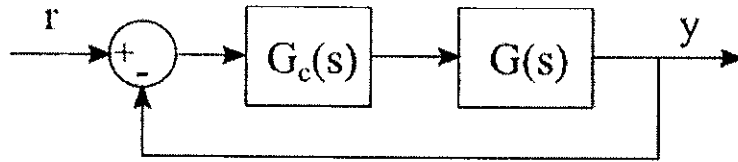


Figure Q4

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1 - e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at - 1 + e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at} - ae^{-bt}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1 - e^{-at} \left(\cos bt + \frac{a}{b} \sin bt \right)$