UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: February 2020

Module Number: IS6301 **Module Name: Mathematical Modelling**

[Three hours]

[Answer **all questions**, each question carries fourteen marks]

Q1.

a) i.) Briefly explain the stages of 'Mathematical Modelling Process'.

ii.) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to each solution? What is the added value in each case?

[3 Marks]

b) The speed, v in m/s, of a train leaving a station can be modelled by the equation:

$v = 6\sqrt{t}$, where t is time in seconds.

i.) Sketch a graph of this information for $0 \leq t \leq 100$.

ii.) The train has stopped at a railway station. Find the time taken for the train to travel a distance of 1500m after leaving the station.

iii.)Calculate the acceleration at the time when the train has travelled 1500m from the station.

[6 Marks]

c) A brine solution of salt flows at a constant rate of 5 L/min into a large tank that initially held 100 L of pure water. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 4 L/min. Assume that the concentration of salt in the brine entering the tank is 0.2 kg/L. Determine the mathematical expression for the above problem and calculate the concentration of salt in the tank after 30 minutes.

[5 Marks]

Q2.

a) i.) Briefly explain the importance of 'Dimensional Analysis' in mathematical modelling of real world problems.

ii.) Clearly state the 'Buckingham Pi Theorem'.

[2 Marks]

b) A cylinder with a diameter, D, floats upright in a liquid as shown in the Figure Q2. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that this frequency is a function of the diameter, D, the mass of the cylinder, m, the liquid density, ρ, and the acceleration due to gravity, g.

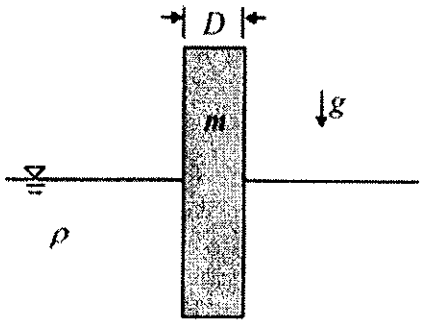

Figure Q2

i.) Using the Buckingham Pi theorem, identify the dimensionless groups (Pi terms) and then derive a relation between the frequency and the four variables.

ii.) If the mass of the cylinder were doubled (assuming the same cylinder material density), by how much would the oscillation frequency change?

[5 Marks]

c) A dietician has to develop a special diet using two foods X and Y. Each packet (containing 30g) of food X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. Using graphical method, determine the number of packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

[7 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of 'Vogel's Approximation Model (*VAM*)' and 'Modified Distribution Method (*MODI*)' used to solve transportation problems.

[2 Marks]

c) A large manufacturing company is closing three of its existing plants and intends to transfer some of its more skilled employees to three plants that will remain open. The number of employees available for transfer from each closing plant is as follows.

Closing Plant	**Transferable Employees**
1	60
2	105
3	70
Total	**235**

The following number of employees can be accommodated at the three plants remaining open.

Open Plants	Employees Demanded
A	45
B	90
C	35
Total	**170**

Each transferred employee will increase product output per day at each plant as shown in the following table. The company wants to transfer employees so as to ensure the maximum increase in product output.

From	To		
	A	**B**	**C**
1	5	8	6
2	10	9	12
3	7	6	8

i.) Find the initial solution using *VAM*.

ii.) Find the total product output increase for the initial feasible solution.

iii.)Solve the problem using *MODI*.

iv.) Are there multiple optimal solutions? Explain. If yes, identify them.

[10 Marks]

Q4.

a) Briefly explain what is meant by,

i.) 'Row reductions' and 'Column reductions' performed in assignment problems.

ii.) Prohibited Assignment.

[2 Marks]

b) Give examples (2 each) for maximization and minimization assignment problems occurring in real world with related to engineering/management.

[2 Marks]

c) A Company employs typists on hourly basis. There are five typists for service and their charges and speeds are shown below. According to an earlier understanding, only one job is given to one typist and is paid for a full hour even if the typist works for a fraction of an hour. The available five jobs are shown below. It was reported that Typist D is unable to perform Job R due to the complex technical nature of that job. Find the least cost required to complete 5 jobs by assigning each typist a different job, by using Hungarian method. If there are multiple optimal solutions, identify the assignments of each typist to different jobs.

Typist	Rate per hour (Rs.)	No. of pages Typed / hour
A	5	12
B	6	14
C	3	8
D	4	10
E	4	11

Job	No. of pages
P	199
Q	177
R	145
S	198
T	178

[10 Marks]

Q5.

a) Find the equilibrium point, if it exists, for the following dynamical system. State whether the equilibrium point stable or unstable.

$$u(n) = 0.5u(n-1) + v(n-1) + 3$$

$$v(n) = 0.4u(n-1) - v(n-1) + 6$$

[2 Marks]

b) Vitamin A is stored primarily in human being's Plasma and Liver. Suppose that 30% of the Vitamin A in the Plasma is filtered out by the Kidneys each day and that 60% of the Vitamin A in the Plasma is absorbed into the Liver each day. Also assume that 10% of the Vitamin A in the Liver is filtered out and that 5% of the Vitamin A in the Liver is absorbed back into the Plasma each day. Suppose you have a daily intake of 3.5mg of Vitamin A, which goes directly into the Plasma.

(Assume that the 1st dose is taken at 1st day night and all conversions of Vitamin A happen during daytime from 2nd day onwards)

i.) Determine a discrete dynamical system for $P(t)$ and $L(t)$, the number of milligrams of Vitamin A in the Plasma and the Liver respectively, after t days of your 1st dose of Vitamin A.

ii.) Find the equilibrium amounts of Vitamin A in the Plasma and Liver.

[6 Marks]

c) Consider the following first-order recurrence relation,

$$X_{n+1} = F(X_n) = X_n^2 - 2a \quad ; \text{where } a \text{ is a constant.}$$

i.) If $a = 0$, find all fixed points and determine their stability.

ii.) For what range of values of a does the recurrence relation has two fixed points?

iii.) If the recurrence relation has two fixed points, for what range of values of a are both unstable?

iv.) If the recurrence relation has a cycle of length 2, such that $X_0 = X_2 = X_4 = \ldots\ldots$ and $X_1 = X_3 = X_5 = \ldots\ldots$, then show that $X_n + X_{n+1} = -1$ for all $n \in \mathbb{N}$.

[6 Marks]