UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: February 2020

Module Number: EE8210 **Module Name: Digital Communication**

[Three Hours]

[Answer **all questions,** each question carries **10 marks]**

Q1 a) When a signal $s_m(t)$ from a signal set is sent, the received signal has the form of

$$r(t) = s_m(t) + w(t)$$

where $w(t)$ represents all unwanted signals added during the transmission. Assume that each signal $s_i(t)$ in the signal set can be represented as a linear combination of set of basis signals and thus it creates Span$\{B\}$. Briefly explain how the orthogonal property among signals is used to approximate the received signal.

[3.0 Marks]

b) i) Determine the signal space representation of four signals $s_i(t)$, $i = 1, 2, 3, 4$, using the basis functions $f_1(t)$ and $f_2(t)$ shown in Figure Q1.

ii) Sketch the signal space diagram and show that this signal set is equivalent to a set of four PSK signals.

[7.0 Marks]

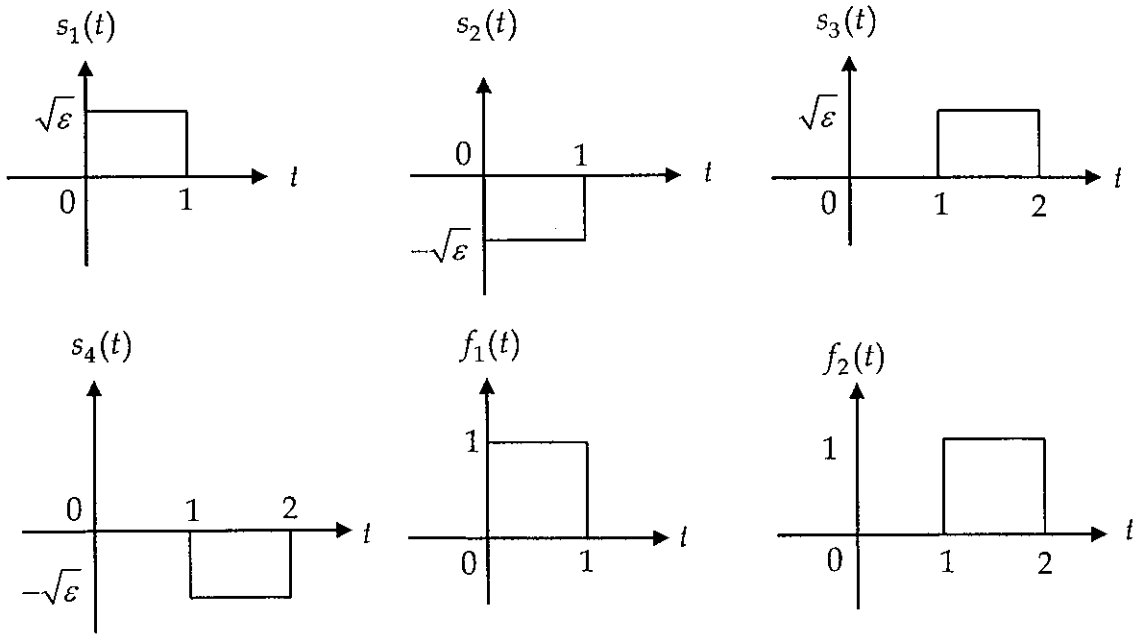


Figure Q1

Q2 a) When simulating a digital communication system in Matlab, it is required to relate the following functions.

$$Q(x) = \mathrm{P}(X > x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{(-\omega^2/2)} d\omega$$

$$erf(x) \; \square \; \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{(-t^2)} dt$$

Show that $Q(y) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{y}{\sqrt{2}}\right)$ for $y = \sqrt{2}x$.

[4.0 Marks]

b) Suppose that the receiver of a digital communication system uses the $X_2 = X_1 + N$ expression where $X_1 \in (-1,1)$ with equal probabilities and N is independent of X_1. The mean and the variance of N are 1 and 0.25 respectively. Furthermore, the receiver makes decisions based on the following criteria.

- If $X_2 \leq \frac{1}{3}$, then the receiver decides that the transmitter has sent "0".
- If $X_2 > \frac{1}{3}$, then the receiver decides that the transmitter has sent "1".

Determine the error probabilities at the receiver in terms of $Q(.)$ function for the following cases.

i) Given that $X_1 = 0$, the receiver decides the transmitter has sent a "1".

ii) Given that $X_1 = 1$, the receiver decides the transmitter has sent a "0".

[6.0 Marks]

Q3 a) A typical M-ary PAM (Pulse Amplitude Modulated) system uses the following amplitudes for M signal points, where M is an even number.

$$-(M-1)A, \; -(M-3)A, \; \ldots\ldots, \; -A, \; A, \; \ldots\ldots, \; (M-3)A, \; (M-1)A$$

i) Determine an expression for the average bit energy for the M-ary PAM system.

ii) Simplify the expression obtained in part i) to determine the probability of symbol error in a binary PAM system using the Maximum Likelihood (ML) detection. Assume that two symbols are equally likely.

iii) Hence, obtain an expression for the overall probability of symbol for the M-ary PAM system by using pairwise comparison. Assume that all symbols are equally likely.

[7.0 Marks]

b) Assuming the maximum-likelihood detection rule, sketch the decision boundaries for the signal constellation diagram shown in Figure Q3.

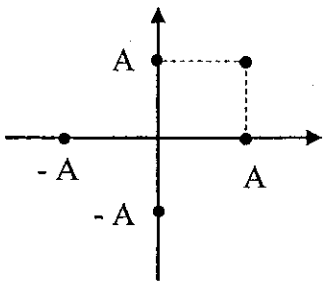


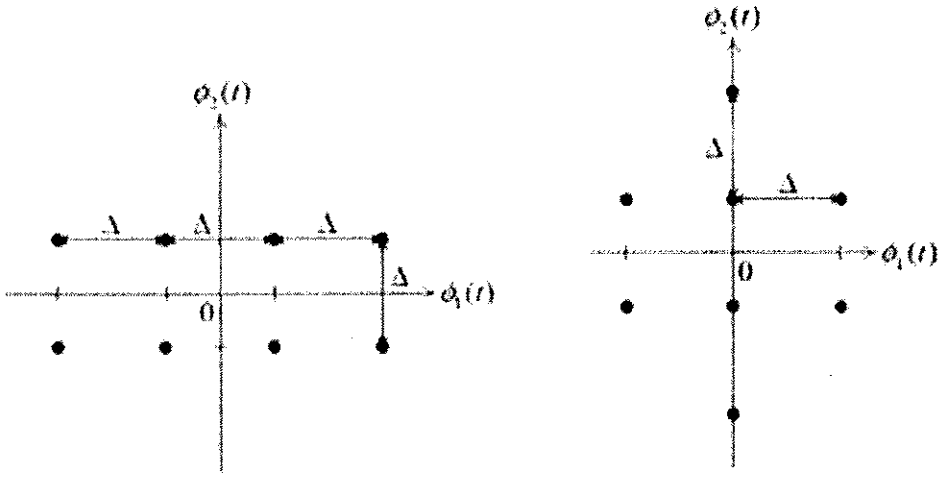
Figure Q3

[3.0 Marks]

Q4 a) Determining the exact probability for symbol error in Phase Shift Keying (PSK) signal constellations is difficult. Explain why.

[2.0 Marks]

b) Figure Q4 shows two signal space diagrams with 8-ary Quadrature Amplitude Modulation (QAM) constellations. Consider that each signal point is equally likely.



Constellation - A Constellation - B

Figure Q4

i) Compute the average signal energies of both constellations, as functions of Δ.

ii) Determine the probability of symbol error for each constellation using the union bound method and the nearest-neighbour approximation method.

iii) Which constellation is the best? Justify your answer.

[8.0 Marks]

Q5 a) In M-ary FSK (Frequency Shift Keying), symbols are selected to be sinusoids with their frequencies selected from a set of M different frequencies $\{f_0, f_1, \ldots\ldots, f_{M-1}\}$. Therefore, the frequency modulated cosine signal for different FSK symbols are changed with the center frequency over the time variable as $f_k = f_c + k\Delta f$. Determine a condition to make two FSK signals that are orthogonal.

Hint:

Consider the orthogonal FSK signals as $Q_k(t) = \sqrt{2}\, p(t)\cos[2\pi f_c t + 2\pi k\Delta f t]$

where $p(t) = \begin{cases} \dfrac{1}{\sqrt{T_s}} & 0 \le t \le T_s \\ 0 & \text{otherwise} \end{cases}$.

[4.0 Marks]

b) Briefly explain the difference between the two reception techniques used for FSK detection. Use block diagrams to support your answer.

[3.0 Marks]

c) The probability of symbol error calculation method of a bipolar PAM system can be used to determine the probability of symbol error in a coherent binary FSK system. Obtain an expression for that by clearly showing the steps.

[3.0 Marks]