University of Ruhuna - Faculty of Medicine

Allied Health Science Degree Programme

First B. Pharm. Part I Examination - August 2017

PH1152 : Mathematics (SEQ)

Time: Two (02) Hours

Each question carries equal marks

Instructions:

- Answer all questions.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.

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1. a) Find the following limits:

(i) $\lim_{x \to -4} \frac{16 - x^2}{4 + x}$,

(ii) $\lim_{x \to 6} \frac{x^2 - 6x}{x^2 - 7x + 6}$,

(iii) $\lim_{x \to \infty} \frac{-6x^4 + x^2 + 1}{2x^4 - x}$.

b) Consider the function $f(x) = 2x^2$. Show that $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 4x$.

c) Suppose a bacterial culture grows in such a way that at time t there are t^3 bacteria. Let $y = t^3$. Find the rate of growth at time t, that is $\frac{dy}{dt}$, **using first principles.** What is the rate of growth at time $t = 10^3$ seconds?

d) Differentiate the following functions with respect to x:

(i) $f(x) = \ln(x^2 + xe^x)$,

(ii) $h(x) = \tan x (\sin x - 5)$,

(iii) $g(x) = \frac{x^2 - 1}{x^2 + 1}$.

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2. a) Consider the function $y = \frac{1}{4}x^4 - x^3 + x^2$.

(i) Find the turning points of this function.

(ii) Identify the above turning points as maxima or minima using the second derivative $\frac{d^2y}{dx^2}$.

Continued.

79

b) A two variable function is given by

$$f(x,y) = \sqrt{1-x^2-y^2}.$$

(i) Find the partial derivatives $\left(\frac{\partial f}{\partial x}\right)_y$, and $\left(\frac{\partial f}{\partial y}\right)_x$.

(ii) Show that the total differential df of f at the point $\left(\frac{2}{3}, \frac{1}{3}\right)$ is given by

$$df = -dx - \frac{1}{2}\,dy.$$

(iii) Verify that

$$\left[\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)_y\right]_x = \left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)_x\right]_y.$$

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3. a) Show that the function $f(x,y) = \sqrt{xy}$ is homogeneous and satisfies the Euler's theorem.

b) Using the substitution $u = 1-4x^2$, evaluate

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

c) Use **integration by parts formula** to show that

$$\int \ln(x+1)\,dx = x\ln(x+1) - x + \ln(x+1) + C;$$

where C is an arbitrary constant.

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4. a) Find partial fractions of $\frac{2}{s^2-1}$.

Writing $\frac{s^2+1}{s^2-1}$ as $1+\frac{2}{s^2-1}$, evaluate $\int_2^3 \frac{s^2+1}{s^2-1}\,ds$.

b) Show, by the **method of separation of variables**, that the solution of the differential equation

$$\frac{dy}{dx} = xe^{2y}$$

can be written as

$$y = -\frac{1}{2}\ln(-x^2+C);$$

where C is an arbitrary constant.

Given the initial condition $y(0) = -1$, find the constant C and write down the solution.

c) Test the differential equation

$$(e^{4x} + 2xy^2)\,dx + (\cos y + 2x^2y)\,dy = 0$$

for exactness. If it is exact, then find its solution.
