UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: October 2019

Module Number: CE7203 **Module Name: Computer Analysis of Structures**

[Three Hours]

[Answer all questions. Each question carries marks as indicated]

Q1. Fig. Q1 shows an idealized plane truss structure. Lengths of all the members are as indicated the figure. AE is a constant for all members. A point load, P, is applied with an angle at 45° to the horizontal axis at the joint D.

a) Using first principles, show that the element flexibility matrix for a bar element is given by $\frac{L}{AE}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ where, L, A, and E have their usual meanings.

[3 Marks]

b) Determine following quantities for the truss structure using matrix flexibility method of analysis.

i) Statical determinacy for the structure

[1 Mark]

ii) Internal forces in all the members and the support reactions

[8 Marks]

iii) Vertical and horizontal displacement at joint D

[3 Marks]

Q2. An idealised two-dimensional frame structure is shown in Fig. Q2. Axial deformations in the members are negligible. The support A has settled by 5mm in vertically downward and a point load of 10 kN has been applied at B as shown in Fig.Q2.

a) Matrix stiffness method is governed by degree of kinetic indeterminacy (KI°). Explain briefly what is kinetic indeterminacy and determine KI° for the structure given in Fig. Q2.

[3 Marks]

b) Using matrix stiffness method, determine the support reactions and nodal deformations (i.e. displacement and rotation) at joint B. All the members have flexural rigidity (EI) of 80x10^3 kNm^2

[12 Marks]

Use stiffness matrix for a two-dimensional beam element with negligible axial deformation as $[k]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$.

Q3. a) Fig. Q3 shows a one dimensional spring assembly. Element numbers are boxed and node numbers are circled. $k_1 = 800$ kN/m, $k_2 = 500$ kN/m and $k_3 = 400$ kN/m and where k_1, k_2 and k_3 are stiffness of the spring element 1, 2 and 3 respectively.

i) Assemble the global stiffness matrix of the system of springs.

[3 Marks]

ii) Determine the displacement at Node 2.

[2 Marks]

iii Determine support reaction at each node.

[2 Marks]

b) Using stiffness equation for 3D element, $[K^n] = \int^{V} [B]^T [D][B] d(vol)$, show that the element stiffness matrix for one dimensional bar element is given by

$$\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

[3 Marks]

Q4. A Pin-jointed 2D truss is pinned support at Node A and roller support at Nodes B and C as shown in Fig Q4. The Young's modulus $E = 220$ GPa for all three elements and cross-section area $A = 6 \times 10^{-4}$ m^2 for element (1) and (2), $6\sqrt{2} \times 10^{-4}$ m^2 for element (3). The truss system is subjected to a force 1500 kN at Node C, as indicated in Fig. Q4.

a) Write the element stiffness matrix of the 3 elements with respect to a selected global coordinate system.

[3 Marks]

b) Determine the global stiffness matrix of the system.

[1 Mark]

c) Define the boundary condition and loading condition for each node.

[2 Marks]

d) Determine the displacement at Nodes B and C.

[3 Marks]

e) Determine support reaction at each node.

[1 Mark]

(Use the stiffness matrix for a 2D-bar element as shown below.)

$$[k^e] = \frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

where $c = Cos\Theta$, $s = Sin\Theta$ and Θ is the anticlockwise angle at node measured from the global X-axis to the local x-axis of the bar element.

Q5. a) Briefly explain the difference between a bar element and a beam element.

[1 Mark]

b) A circular beam **ABC** fixed at Node **A**, roller support at Node **B** as shown in Fig Q5. The Young's modulus of the beam is $E = 210$ GPa. Consider uniform load applied for beam BC as 24 kN/m and a clockwise moment 12 kNm at Node B.

i) Determine a minimum diameter of solid circular cross section for beam, such that the maximum deflection at node C does not exceed 150 mm.

[8 Marks]

ii) Compute the nodal displacements and rotations.

[1 Mark]

(Ignore the axial effect and use the element stiffness matrix for a 2-D beam element given at the end of Question 2)

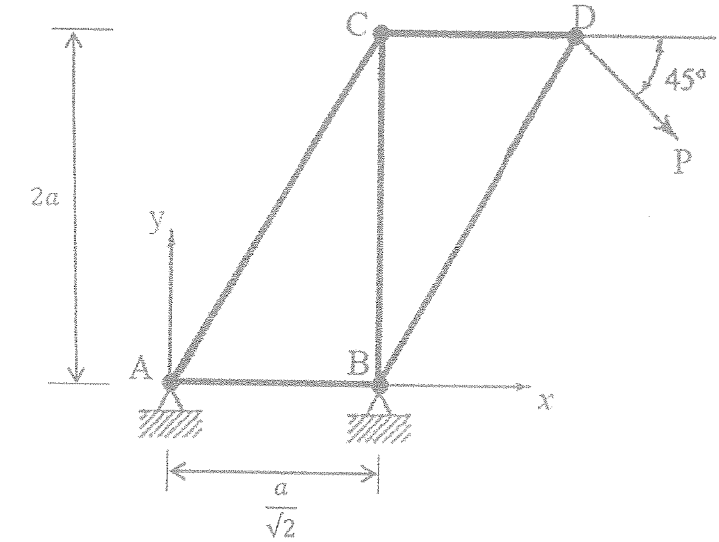

Fig. Q1

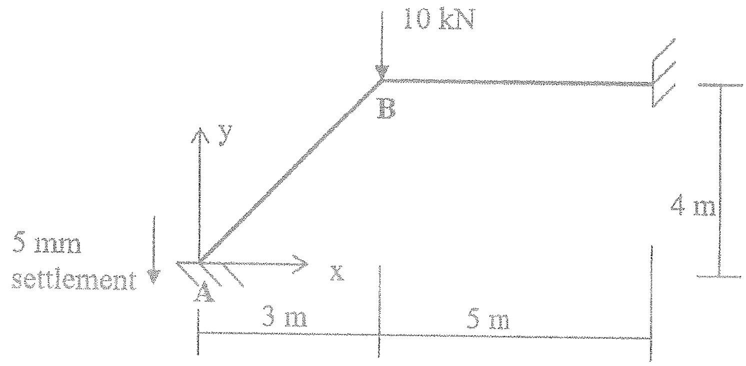

Fig. Q2

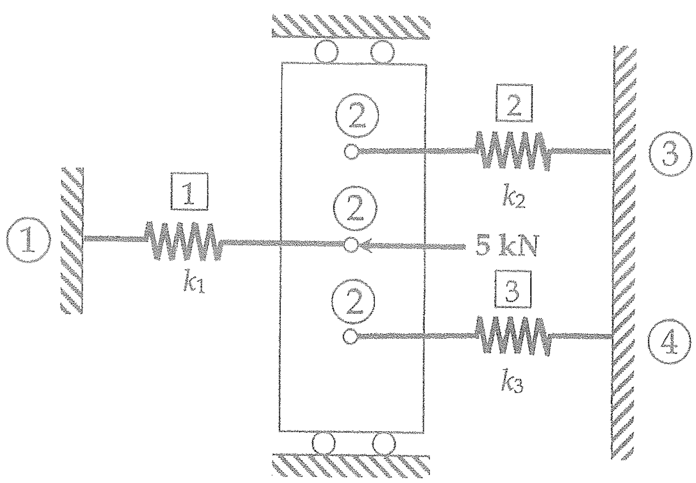

Fig. Q3

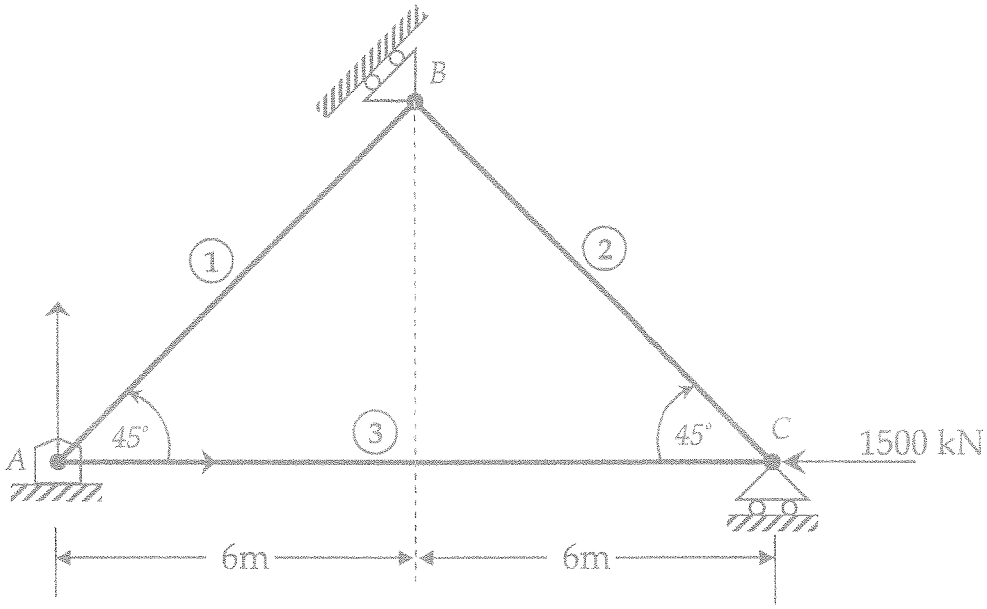

Fig. Q4

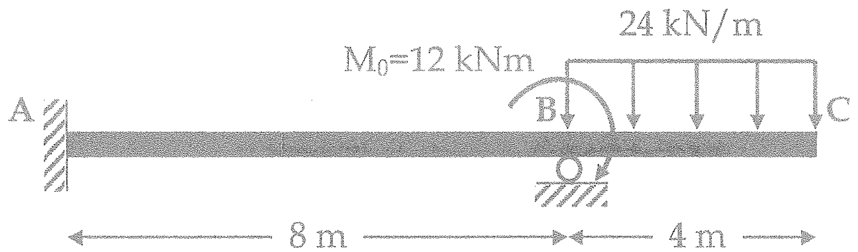

Fig. Q5