UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: October 2019

Module Number: ME 7303 **Module Name: Solid Mechanics**

[Three Hours]

[Answer **all questions**, each question carries **12** marks]

Q1. a) Consider a thin spherical shell which is subjected to an internal fluid pressure. Show that the volumetric strain of spherical shell due to internal fluid pressure is equal to three times of its radial strain and hence show that change in volume of the spherical shell due to internal fluid pressure is given by,

$$\frac{P\pi d^4(1-2\nu)}{8Et}$$

where,

P - internal fluid pressure
d - internal diameter of the spherical shell
E - modulus of elasticity
ν - poisson's ratio
t - thickness of the wall of the spherical shell.

[6.0 Marks]

b) i) A spherical shell of internal diameter 85 cm and of thickness 11 mm is subjected to an internal fluid pressure of 2.3 N/mm². Determine the increase in diameter and changing volume of spherical shell due to internal fluid pressure. Take modulus of elasticity as 250 GPa, and poisson's ratio as 0.3.

[4.0 Marks]

ii) Calculate the increase in diameter and changing volume of spherical shell, if the joint efficiency of the spherical shell is 90%.

[1.0 Mark]

c) Briefly explain two industrial applications of thin spherical shells.

[1.0 Mark]

Q2. Consider a thin disc of uniform thickness with inner and outer radii r_1 and r_2, respectively, rotates about its axis at ω angular velocity. Take the density of disc material as ρ and poisson's ratio as ν.

a) Show that the radial stress (σ_r) and circumferential stress (σ_c) of the rotating disc at a radius r are given by,

$$\sigma_r = \frac{\rho\omega^2(3+\nu)}{8}\left[r_1^2 + r_2^2 - \frac{r_1^2 r_2^2}{r^2} - r^2\right]$$

$$\sigma_c = \frac{\rho\omega^2(3+\nu)}{8}[r_1^2 + r_2^2 + \frac{r_1^2 r_2^2}{r^2} - \frac{(1+3\nu)r^2}{(3+\nu)}]$$

[5.0 Marks]

b) Derive expressions for the maximum and minimum circumferential stresses of the disc.

[4.0 Marks]

c) Derive expression for the maximum radial stress of the disc.

[2.0 Marks]

d) Using above derived expressions, sketch circumferential stress and radial stress distribution across the radius.

[1.0 Mark]

Q3. a) A steel shear coupling in a metal working process is 50 mm diameter. It is subjected to a torque of 900 Nm which is known to have shear yielding in the shaft. Determine the radial depth to which plasticity has penetrated if yielding shear stress is 100 MN/m^2.

[5.0 Marks]

b) A hollow circular shaft of outer and inner diameters of 25 cm and 15 cm, respectively, is subjected to 50 MN/m^2 yielding shear stress. Find the maximum torque which the shaft can transmit under fully elastic condition.

[4.0 Marks]

c) Consider a thin cylinder of internal diameter 2.5 m containing a fluid at an internal pressure of 3 N/mm^2. Determine the maximum thickness of the cylinder if,

i) The longitudinal stress is not exceeding 45 N/mm^2.

[1.5 Marks]

ii) The circumferential stress is not exceeding 60 N/mm^2.

[1.5 Marks]

Q4. a) Part of a landing gear of an aeroplane is subjected to the following direct stresses in the x, y and z directions: 195 MPa, 35 MPa and -250 MPa respectively. Due to different landing conditions there are also possibilities of two shear stresses present, one related to xz plane with a value of 30 MPa and another related to yz plane with a value of 65 MPa.

i) Draw the elemental cube showing the stresses acting.

[2.0 Marks]

ii) Draw separate Mohr's circles associated with xy, xz, and yz planes on a same graph-sheet.
(**Clue:** No need to consider shear stresses)

[3.0 Marks]

iii) Using this information given above, determine the largest compressive stress associated with the system.
(**Clue**: Need to consider shear stresses)

[3.0 Marks]

iv) Determine the orientation of this compressive stress (Q4. a) iii)) with respect to the xyz co-ordinates system and make a sketch showing the direction.

[2.0 Marks]

b) If the yield strength of the material in tension is 630 MPa and the material follows the von Mises yield criteriondetermine the factor of safety associated with this point in the landing gear assuming the other two principal stresses are 49 MPa and 197 MPa.

(**Clue**: $\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$)

[2.0 Marks]

Q5. In order to verify the behavior, both finite element analysis (predicted) and strain gauge technique (experimental) were used to evaluate a design. The output from the finite element model was used to determine the principal stress values at a position of interest.

Further confirmation was achieved using a strain gauge rosette consisting of three gauges in the pattern shown in Figure Q5 bonded to the surface in the region of the fillet at an angle of 15° to the bending axis. The gauges had a gauge length of 2 mm and bonded using an epoxy adhesive. The output results under the maximum load condition for the three gauges are given below.

$\varepsilon_x = 2630 \times 10^{-6}$ mm/mm (0°)

$\varepsilon_{xy} = 1613 \times 10^{-6}$ mm/mm (45°)

$\varepsilon_y = -1924 \times 10^{-6}$ mm/mm (90°)

a) Draw the Mohr's circles associated with the strain system.

[4.0 Marks]

b) Determine the maximum strain obtained.

[2.0 Marks]

c) Determine the strain in implant axis.

[2.0 Marks]

d) Determine the shear strain in the plane of xy.

[2.0 Marks]

e) While comparing predicted and experimental stresses, there was a difference between the two results. Explain why and where the main source of error is likely to occur.

[2.0 Marks]

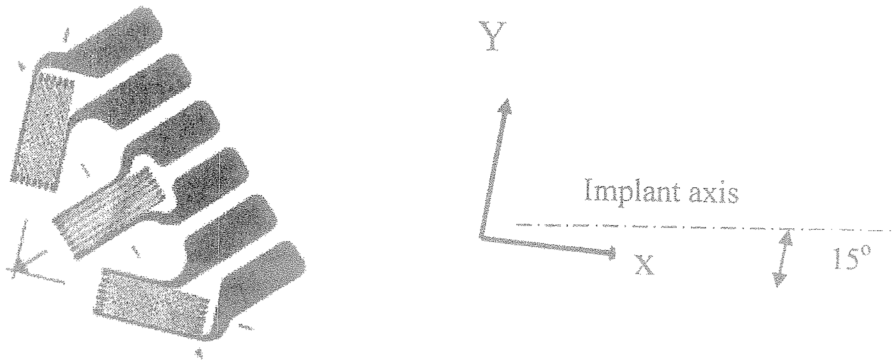

Figure Q5