



UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: August 2018

Module Number: IS1401

Module Name: Mathematical Fundamentals for Engineers

[Three hours]

[Answer all questions, each question carries fourteen marks]

Q1. a) i.) Briefly explain what is meant by the modulus and the argument of a complex number $z = x + iy$, where x and y are real numbers.

ii.) Find the modulus and principal argument of

$$1 - \cos \alpha + i \sin \alpha, \text{ where } \alpha \in \left[0, \frac{\pi}{4}\right].$$

[3 Marks]

b) i.) State the De Moivre's Theorem, and prove the theorem only for the positive integers.

ii.) Simplify $\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)^{1/4}$ by using De Moivre's theorem, and express the result in the form $a + ib$, where a and b are real constants to be determined.

[3 Marks]

c) i.) Find all the roots of $Z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{1/4}$ and represent these roots in the complex plane.

ii.) Determine the real and imaginary parts of $\log(x + iy)$.

[8 Marks]

Q2. a) Briefly explain the following types of matrices, and give an example for each type.

- i.) Diagonal matrix
- ii.) Skew symmetric matrix
- iii.) Hermitian matrix
- iv.) Singular matrix

[2 Marks]

b) i.) If the matrices $\begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & \frac{1}{a} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & -\frac{1}{a} \end{bmatrix}$ constitute an inverse pair, the

determine the value of a .

ii.) For matrices $A = \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -20 & -8 \\ 8 & 4 \end{bmatrix}$, show that $A^{-1} + B = I$, where I is the 2×2 identity matrix.

iii.) Let A and B be two non-singular matrices of the same order and A^{-1} and B^{-1} are the inverse matrices of A and B respectively. Show that $(AB)^{-1} = B^{-1}A^{-1}$.

[5 Marks]

c) i.) Briefly explain what is meant by 'Minor' and 'Cofactor' of a matrix.

ii.) Find the minors and cofactors of the elements a_{11} , a_{12} and a_{13} in the matrix which determinant is denoted by

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Show that if elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

[3 Marks]

d) Find the value of k , such that the system of equations

$$4x_1 + 9x_2 + x_3 = 0$$

$$kx_1 + 3x_2 + kx_3 = 0$$

$$x_1 + 4x_2 + 2x_3 = 0$$

has infinite number of solutions. Hence, find the solution of the system.

[4 Marks]

Q3. a) i.) Briefly explain what is meant by 'Unit vector' and 'Position vector'.

ii.) Show that the three points, whose position vectors are $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $(-7\mathbf{j} + 10\mathbf{k})$ are collinear.

ii.) Prove by using vectors that the line joining the mid-points of two sides of a triangle is parallel and half to the third side.

[5 Marks]

b) A rigid body is spinning with angular velocity 6 radians/sec about an axis OR , where position vector of R is $(2\underline{i} - 2\underline{j} + \underline{k})$ and O is the origin. Find the velocity of the point $(3\underline{i} + 2\underline{j} - \underline{k})$ on the body.

[3 Marks]

c) i.) Show that $\text{div curl } A = 0$, where $A = A_1\underline{i} + A_2\underline{j} + A_3\underline{k}$.

ii.) For the function $\phi(x, y) = \frac{x}{x^2 + y^2}$, find the magnitude of the directional derivative $(\nabla \phi)$ along a line making an angle 30° with the positive x -axis at $(0, 2)$.

[6 Marks]

Q4. a) i.) Explain what is meant by the function ' $f(x)$ is a one to one'.

ii.) Show that if $f(x)$ and $g(x)$ are one to one functions, $(f \circ g)(x)$ is a one to one function.

[2 Marks]

b) i.) Explain what is meant by the function

I) $f(x)$ is continuous at $a \in \mathbb{R}$.

II) $f(x)$ is differentiable at $a \in \mathbb{R}$.

ii.) Show that, if $f(x)$ is finitely differentiable at $a \in \mathbb{R}$, $f(x)$ is continuous at a . Is converse of the above statement true?. Explain your answer.

iii.) Discuss the continuity of the function

$$\begin{cases} 2x + 1; & x < -2 \\ x - 2; & -2 \leq x < 1 \\ 3x - 1; & 1 \leq x \end{cases}$$

iv.) Sketch the graph of $y = |x - 1| - |2x + 1| + |x + 3|$

[10 Marks]

c) Evaluate the following limits

i.) $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$

ii.) $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$

[2 Marks]

Q5. a) i.) State the Rolle's theorem

ii.) Show that the equation $2x^3 - 3x^2 + 6x + k = 0$ has no two real roots between 0 and 1.

[4 Marks]

b) i.) State and prove the mean value theorem

- ii.) If $f(x)$ and $g(x)$ are two functions continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) , show that there exist $c \in (a, b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

[3 Marks]

- c) i.) State and prove the Euler's theorem on homogeneous function of two variables.

- ii.) If

$$u = \cos^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$$

Show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -2 \cot u$$

- iii) Let $u = x^2 + y^2$, where $x = r \cos \theta$ and $y = r \sin \theta$.

Show that

$$\left(\frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta} \right)^2 = 4u$$

[7 Marks]