UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: August 2018

Module Number: IS1402 **Module Name: Mathematical Fundamentals for Engineers**

[Three hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) i.) Briefly explain what is meant by the modulus and the argument of a complex number $z = x + iy$, where x and y are real numbers.

ii.) Find the modulus and principal argument of

$$1 - \cos\alpha + i\sin\alpha \ , \ \text{where} \ \ \alpha \in \left[0, \frac{\pi}{4}\right].$$

[3 Marks]

b) i.) State the De Moivre's Theorem, and prove the theorem only for the positive integers.

ii.) Simplify $\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{1/4}$ by using De Moivre's theorem, and express the result in the form $a + ib$, where a and b are real constants to be determined.

[3 Marks]

c) i.) Express $\dfrac{(\cos 3\theta + i\sin 3\theta)^5.(\cos 4\theta - i\sin 4\theta)^4}{(\cos 5\theta - i\sin 5\theta)^3.(\cos 6\theta + i\sin 6\theta)^2}$ in the form $x + iy$.

ii.) Find all the roots of $Z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{1/4}$ and represent these roots in the complex plane.

[6 Marks]

Q2. a) Briefly explain the following types of matrices, and give an example for each type.

i.) Diagonal matrix
ii.) Skew symmetric matrix
iii.) Hermitian matrix
iv.) Singular matrix

[2 Marks]

b) i.) If the matrices $\begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & \frac{1}{a} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & -\frac{1}{a} \end{bmatrix}$ constitute an inverse pair, then determine the value of a.

ii.) For matrices $A = \begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -20 & -8 \\ 8 & 4 \end{bmatrix}$, show that $A^{-3} + B = I$, where I is the 2×2 identity matrix.

[3 Marks]

c) i.) Briefly explain what is meant by 'Minor' and 'Cofactor' of a matrix.

ii.) Find the minors and cofactors of the elements a_{11}, a_{12} and a_{13} in the matrix which determinant is denoted by

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Show that if elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

[3 Marks]

d) Find the value of k, such that the system of equations

$$\begin{aligned} 4x_1 + 9x_2 + x_3 &= 0 \\ kx_1 + 3x_2 + kx_3 &= 0 \\ x_1 + 4x_2 + 2x_3 &= 0 \end{aligned}$$

has infinite number of solutions. Hence, find the solution of the system.

[4 Marks]

Q3. a) i.) Show that the three points, whose position vectors are $(2\underline{i} + 3\underline{j} - 4\underline{k})$, $(\underline{i} - 2\underline{j} + 3\underline{k})$ and $(-7\underline{j} + 10\underline{k})$ are collinear.

ii.) Prove by using vectors that the line joining the mid-points of two sides of a triangle is parallel and half to the third side.

[3 Marks]

b) A rigid body is spinning with angular velocity 6 radians/sec about an axis OR, where position vector of R is $\left(2\underline{i} - 2\underline{j} + \underline{k}\right)$ and O is the origin. Find the velocity of the point $\left(3\underline{i} + 2\underline{j} - \underline{k}\right)$ on the body.

[3 Marks]

c) i.) Show that $div\ curl\ A = 0$, where $A = A_1\underline{i} + A_2\underline{j} + A_3\underline{k}$.

ii.) For the function $\phi(x, y) = \dfrac{x}{x^2 + y^2}$, find the magnitude of the directional derivative ($\nabla\phi$) along a line making an angle 30° with the positive x-axis at (0,2).

[6 Marks]

Q4. a) i.) Explain what is meant by the function '$f(x)$ is a one to one'.

ii.) Show that if $f(x)$ and $g(x)$ are one to one functions, $(f \circ g)(x)$ is a one to one function.

[2 Marks]

b) i.) Explain what is meant by the function

I) $f(x)$ is continuous at $a \in \mathbb{R}$.

II) $f(x)$ is differentiable at $a \in \mathbb{R}$.

ii.) Show that, if $f(x)$ has a finite derivative at $a \in \mathbb{R}$, $f(x)$ is continuous at a.

iii.)Write down an example for a real function $f(x)$, which satisfies each of the following condition.

I) Limit of $f(x)$ exists for all $x \in \mathbb{R}$, but $f(x)$ is continuous only for $x \in \mathbb{R} - \mathbb{Z}$.

II) $f(x)$ is not differentiable only at $x = 1$ and $x = 2$.

iv.) Sketch the graph of $y = |x - 1| - |2x + 1| + |x + 3|$

[8 Marks]

c) Evaluate the following limits

i.) $\displaystyle\lim_{x\to 0} \frac{x \sin x}{1 - \cos x}$ ii.) $\displaystyle\lim_{x\to 1} \frac{\ln x}{x^2 - 1}$

[2 Marks]

Q5. a) i.) State and prove the Rolle's theorem.

ii.) Show that the equation $2x^3 - 3x^2 + 6x + k = 0$ has no two distinct real roots between 0 and 1.

[5 Marks]

b) i.) State and prove the mean value theorem.

ii.) If $f(x)$ and $g(x)$ are two functions continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) and $g'(x) \neq 0$ for all $x \in (a, b)$, show that there exist $c \in (a, b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}.$$

[3 Marks]

c) i.) State and prove the Euler's theorem on homogeneous function of two variables.

ii.) Let

$$u = \cos^{-1}\left(\frac{x^3 + y^3}{x - y}\right).$$

Show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -2\cot u.$$

[4 Marks]