UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: Aug/Sep 2018

Module Number: ME 3303 **Module Name: Modelling of Dynamic Systems**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

A partial table of Laplace transformation pairs is given on page 6. You may make additional assumptions where necessary, but clearly state them in your answers.

Q1 a) Use the Laplace transform to solve the following initial value problem.

$$\dot{y}(t)+4y(t)=g(t);\;\; y(0)=2,$$

$$where\;\; g(t)=\begin{cases}0, & 0\le t<1\\ 12, & 1\le t<3\\ 0, & t\ge 3\end{cases}$$

[4.0 Marks]

b) Use the Convolution Integral Theorem to find the inverse Laplace transform of the following transfer function.

$$H(s)=\frac{1}{(s^2+a^2)^2};\; where\; a=\; constant.$$

[3.0 Marks]

c) A 16 kg weight is attached to a spring with a spring constant of 2 Nm^{-1}. The damping effect is negligible. The weight is released from the rest at 3m below the equilibrium position. At t = 2π second, it is struck with a hammer, providing an impulse of 4 Ns. This situation is modeled by the initial value problem,

$$\frac{16}{32}\ddot{y}(t)+2y(t)=4\delta(t-2\pi);\quad y(0)=3,\; \dot{y}(0)=0.$$

Determine the displacement function $y(t)$ of the weight.

[5.0 Marks]

Q2 a) Figure Q2(a) shows a thermal model representing the heat exchange between a laboratory vacuum flask and the environment. The differential equation describing the dynamics of the fluid temperature T_c is found by,

$$R_tC_t\frac{dT_c(t)}{dt}+T_c(t)=T_{amb}(t);$$ where C_t = Thermal capacitance of the fluid,

R_t = Thermal resistance of the wall, T_{amb} = Ambient temperature of the external environment.

i) If the vacuum flask is exposed to a constant ambient temperature, obtain an expression for the fluid temperature in terms of steady-state response, transient response, and, initial condition response.

ii) Obtain the time constant of the above system.

iii) Briefly explain the effect of the initial conditions on the total response obtained in the part i)

[7.0 Marks]

Q2 is continued to next page...

b) A unit step response of an open loop plant is shown in the Figure Q2(b).
   i) Obtain the DC gain (DCG) of the plant.
   ii) Propose a method to obtain the above response with unity DC gain.

[2.0 Marks]

c) Describe why second order systems are the most popular in control systems literature and also in industrial control system implementations.

[1.0 Mark]

d) A measurement conducted on a servomechanism shows that the time domain response of the system to be $y(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$ when subjected to a unit step input. Obtain the expression for the system transfer function, $G(s) = Y(s)/R(s)$; where $R(s)$ is the unit step input.

[2.0 Marks]

Q3 a) Following Matlab code is used to obtain the Root Locus [Figure Q3(a)] of a servo control system.

```
close all;
clear all;
num=[1 6]; den=[1 3 2];
G=tf(num,den);
rlocus(G);
grid on;
```

Show the closed loop servo plant with a simple feedback gain (K) in a block diagram.

[2.0 Marks]

b) With the help of the Root Locus [Figure Q3(a)], determine
   i) number of asymptote(s) and asymptote angle(s),
   ii) asymptote intersection point(s),
   iii) break away/in point(s).

[3.0 Marks]

c) Read from the Root Locus, the values of K for which the response is,
   i) critically damped and,
   ii) most oscillatory damped.

[2.0 Marks]

d) Comment on the value(s) of feedback gain (K) which the plant can show an unstable behavior?

[1.0 Mark]

e) *'Root Locus Design method can be used to locate poles at some desired locations. However, it is not possible to locate poles arbitrarily'.* Briefly explain the above statements giving reasons.

[1.0 Mark]

f) The above servo control system is approximated to a generic second order system eliminating the existing zero. If the plant demands 5% peak overshoot and 2s settling time, show that the desired poles are located outside the Root Locus of the new system.

[3.0 Marks]

Useful: For under-damped generic second order systems, The transfer function

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \text{ Peak overshoot } PO = e^{\frac{-\varsigma\pi}{\sqrt{1-\varsigma^2}}}, \text{ Settling time } t_s = \frac{4.6}{\varsigma\omega_n}.$$

Q4 a) An inverted pendulum system mounted on a motor-driven cart is shown in the Figure Q4(a). This is a model of the attitude control of a space booster on takeoff. (The objective of the attitude control problem is to keep the space booster in a vertical position). The inverted pendulum is unstable and it may fall over any time in any direction unless a suitable control force $u(t)$ is applied. The angle of the rod from the vertical line is considered to be $\theta(t)$. The mathematical model of this system is explained by,

$$(M+m)\ddot{x}(t)+ml\ddot{\theta}(t)=u(t),$$

$$ml^2\ddot{\theta}(t)+ml\ddot{x}(t)=mgl\theta(t).$$

Obtain the state space model of the system in terms of defined state variables x_1, x_2, x_3, and, x_4.

[6.0 Marks]

b) An approximation of the pendulum system is presented in the form of the state equation, $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$ and,

the output equation, $y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+[0]u$.

i) Obtain the transfer function of the pendulum system.

ii) Develop a Matlab code to obtain the same answer in part i).

[6.0 Marks]

Q5 a) Figure Q5(a) graphically represents three stability criteria of Lyapunov Stability in an autonomous nonlinear dynamical system. The trajectories start at the initial state $x(0)$ and converge to the equilibrium point $\bar{X}$ at the end. X_1 and X_2 represent state variables and, δ, γ, ε are noted for the region of attractions.

i) If $\bar{X}=0$, identify A, B, C figures for stable, asymptotically stable, and, unstable situations.

ii) Briefly explain the concept of "exponential stability" based on the above stability criteria.

[3.0 Marks]

b) Briefly explain how Lyapunov's first method (indirect method) can be used to express the stability of non-linear time-invariant systems.

[1.0 Mark]

c) Determine the stability of the equilibrium of the mechanical system at the origin, $m\ddot{y}(t)+b\dot{y}(t)+k_1y(t)+k_2y(t)^3=f(t)$ using Lyapunov's 2nd method. m, b, k_1, k_2 represent constants.

Hint: Use Lyapunov's linearized technique to investigate the equilibrium with $f(t)=0$.

[4.0 Marks]

d) Prove that, in a linear time-invariant system, $\dot{x}=Ax$ is asymptotically stable if and only if for any positive definite matrix Q there exists a positive definite symmetric solution P to the Lyapunov equation, $A^TP+PA=-Q$.

[2.0 Marks]

e) Determine the stability of the system with state matrix $A=\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}$ using the Lyapunov equation with $Q=I$, where I = Identiy matrix.

[2.0 Marks]

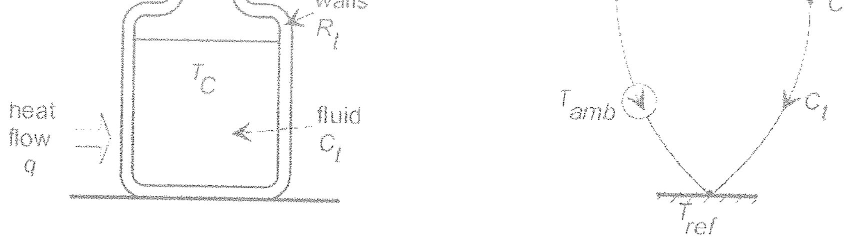

Figure Q2(a): Thermal model representing the heat exchange between a laboratory vacuum flask and the environment

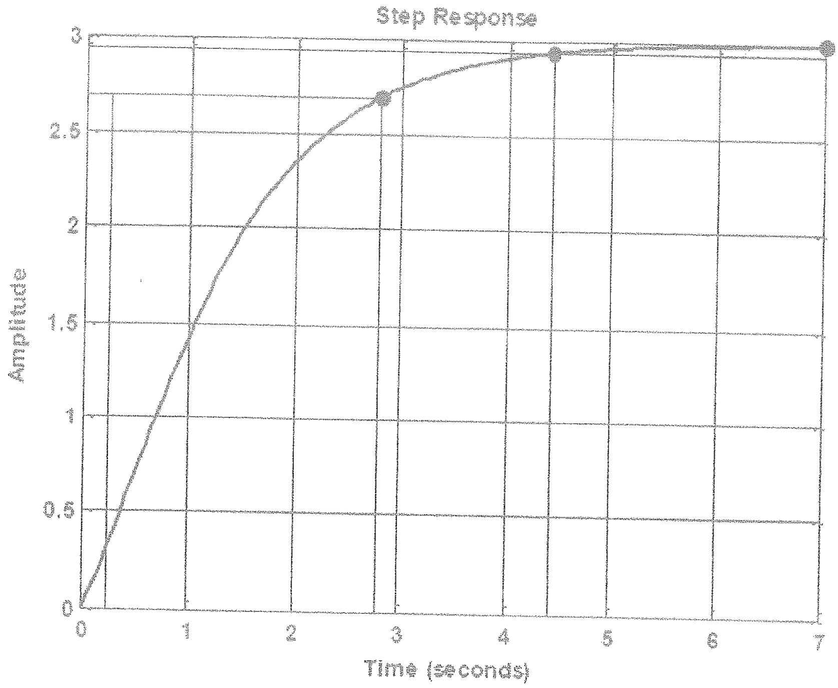

Figure Q2(b): Step response of the plant

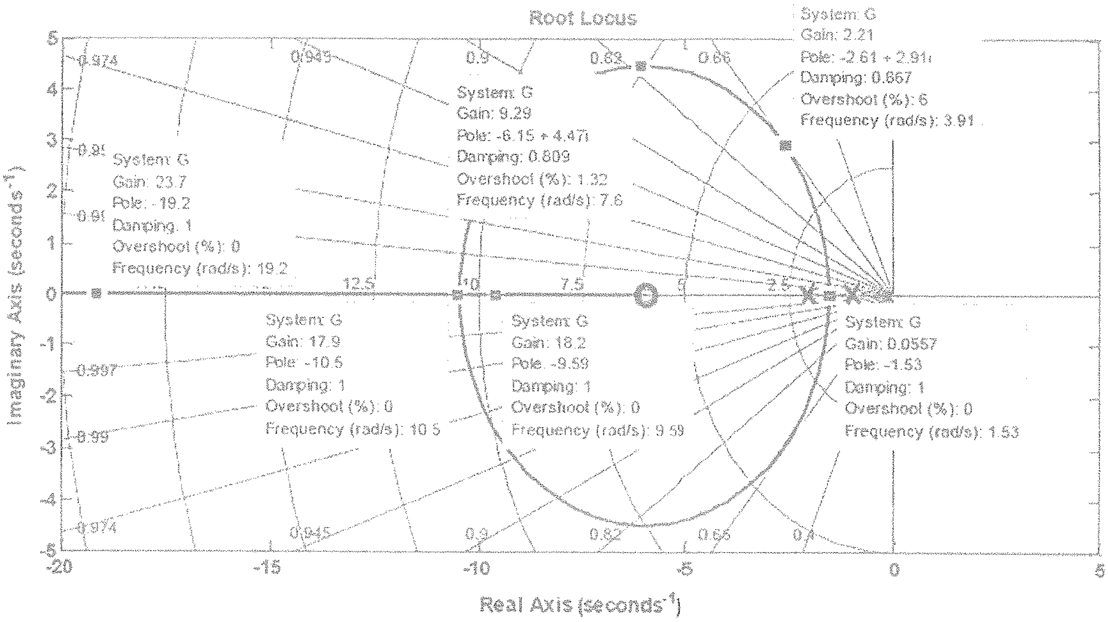

Figure Q3(a): Root Locus of the plant

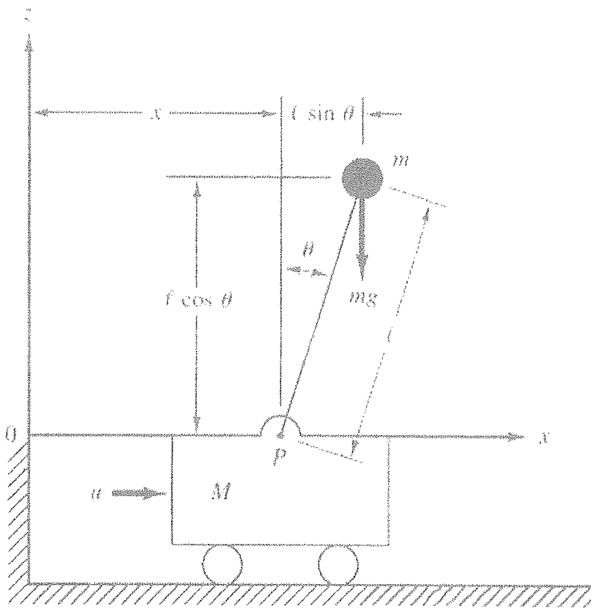

Figure Q4(a): Inverted-pendulum system.

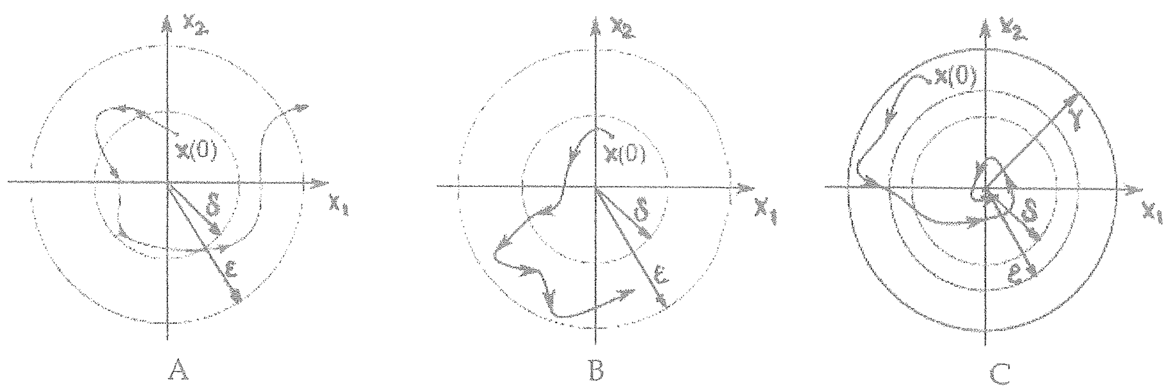

Figure Q5(a): Lyapunov's stability of an equilibrium state

Table of Laplace transform pairs

$f(t)$	$F(s)$
step	$\frac{1}{s}$
ramp, t	$\frac{1}{s^2}$
impulse	1
dirac delta function, $\delta(t-c); c \ge 0$	e^{-cs}
$h(t-a)$	$\frac{e^{-as}}{s}$
$h(t-a)\ g(t-a)$	$e^{-as}G(s)$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\omega_0 \sqrt{1-\zeta^2} t)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
$1-\frac{\omega_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\omega_0 \sqrt{1-\zeta^2} t+\phi)$, *Where*, $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$
$\frac{d(f(t))}{dt}$	$sF(s)-f(0)$
$\frac{d^2(f(t))}{dt^2}$	$s^2F(s)-sf(0)-\dot{f}(0)$
$\int f(t)dt$	$\frac{1}{s}F(s)+\frac{1}{s}\left[\int f(t)dt\right]_{t=0}$
$f(t-\alpha)$	$e^{-\alpha s}F(s)$ *with* $f(t-\alpha)=0, t \le \alpha$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$
$t^n f(t); \quad n=1,2,3$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$f(t/a)$	$aF(as)$
Convolution Intergral; $(f_1 * f_2)(t) = \int_0^t f_1(t-\tau) f_2(\tau) d\tau$	$F_1(s)F_2(s)$