UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: August 2018

Module Number: CE 5204 **Module Name: Structural Analysis III**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) Discuss the near collapse behaviour of a square slab, which is simply supported in all four edges and subjected to a uniformly distributed load of increasing intensity.

[2 Marks]

b) What are the parameters that required in strength assessment of reinforced concrete slabs using yield line method?

[2 Marks]

c) A regular polygonal slab of 'n' sides, which is isotropically reinforced and simply supported along the edges. The perimeter length of the slab was found as 'L'. The slab carries a uniformly distributed load of intensity 'q' per unit area.

i) Draw a possible yield line pattern at collapse.

ii) Determine the corresponding collapse load, assuming the yield moment per unit length of slab is 'm'.

[8 Marks]

Q2. a) Compare load resistance mechanisms of one dimensional flexural member with that of two dimensional flexural member.

[2 Marks]

b) A thin rectangular plate of side dimensions 'a', 'b' and thickness 'h' is shown in Figure Q2. The plate is simply supported along all four edges. It is subjected to a vertical downward load of intensity,

$$p(x,y) = p_o sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}$$

Where, p_o is a constant and the m ,n are positive integers.

i) Assume a trail solution for displacement and show that the trail solution satisfies the relevant displacement and boundary conditions.

[2 Marks]

ii) Determine deflection of the plate. Hence, determine bending moment on the plates.

[3 Marks]

iii) Determine the minimum thickness of the plate for the maximum displacement 0.5 mm when m=3, n=1, a=b=1 m and p_o=150 kPa. The Young's modulus (E) and the poison's ratio (v) of the plate material are

200 GPa and 0.3, respectively.

Governing equation and the equations for bending moments and shear forces (with usual notations and sign convention) are given by

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{q}{D} \qquad M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)$$

$$Q_x=-D\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x \partial y^2}\right) \qquad Q_y=-D\left(\frac{\partial^3 w}{\partial x^2 \partial y}+\frac{\partial^3 w}{\partial y^3}\right)$$

where ,

$$D=\frac{Et^3}{12(1-\nu^2)}$$

[5 Marks]

Q3. a) What are advantages and disadvantages of thin circular plate over thin rectangular plates?

[2 Marks]

b) A thin circular plate of radius a, uniform thickness t, Young's modulus E and poison's ratio ν, is fixed along the edge $r=a$. The plate is subjected to lateral downward pressure which varies according to formula, $p(r)=p_o\frac{r}{a}$

i) Show that the shear force per unit length at r distance from center of the plate can be expressed as $Q=p_o\frac{r^2}{3a}$.

[2 Marks]

ii) Determine an expression for deflection of the plate. Hence, determine the maximum deflection of the plate.

[3 Marks]

iii) This plate covers a circular opening having a radius of 200 mm. The thickness of the plate is 10 mm and it is fixed at outer edge. The E and ν of the plate material are 200 GPa and 0.3, respectively.
Determine the maximum value of p_o when the plate deflection is limited to 1 mm.
State only the steps of determining the maximum permissible value for p_o when safety factor of failure is 2.0. The allowable strength of plate material is noted as $\sigma_{allowable}$.

Governing equation and the equation for the radial moment of circular plate (with usual notations and sign convention) are given by

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\frac{dw}{dr})\right]=\frac{Q}{D} \quad M_r=-D\left(\frac{d^2 w}{dr^2}+\frac{\nu}{r}\frac{dw}{dr}\right) \quad M_t=-D\left(\frac{1}{r}\frac{dw}{dr}+\nu\frac{d^2 w}{dr^2}\right)$$

where

$$D=\frac{Et^3}{12(1-\nu^2)}$$

[5 Marks]

Q4. a) Show that the membrane stresses in a cylindrical shell (*with usual notations and sign convention*) are given by

$$\frac{N_x}{\partial x}+\frac{1}{R}\frac{\partial N_{\phi x}}{\partial \phi}+X=0 \qquad \frac{1}{R}\frac{\partial N_{\phi}}{\partial \phi}+\frac{\partial N_{x\phi}}{\partial x}+Y=0 \qquad \frac{N_{\phi}}{R}+Z=0$$

[6 Marks]

b) A semi-circular cylindrical shell made of thin steel sheets is proposed to use as a cantilever roof of a cafeteria as shown in Figure Q4. The length and the radius of the roof are *L* and r, respectively. Assume that the load acting on the shell is its self weight of '*q*' per unit surface area.
Based on the membrane theory obtained in Q4 Part (a), determine membrane stress resultant in the roof shell structure. Clearly state assumptions you made.

[6 Marks]

Q5 a) Discuss with examples the application of spherical dome in civil engineering structure

[2 Marks]

b) Determine the stress resultants in the spherical dome structure shown in Figure Q5.

Assume that the membrane stresses in a spherical shell (with usual notations and sign convention) are given by

$$\frac{N_{\phi}}{r_1}+\frac{N_{\theta}}{r_2}=P_r \qquad P_{\phi}rr_1-r_1N_{\theta}Cos\phi+\frac{\partial(rN\phi)}{\partial\phi}=0$$

[6 Marks]

c) Determine the forces developed at the horizontal ring beams.

[4 Marks]

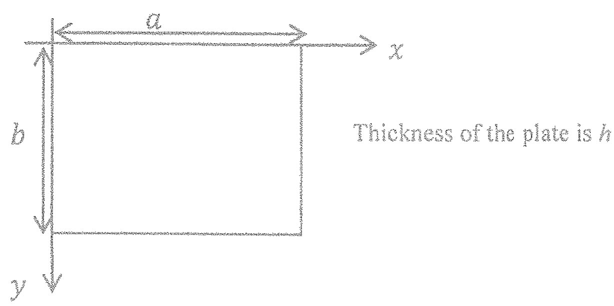

Figure Q2

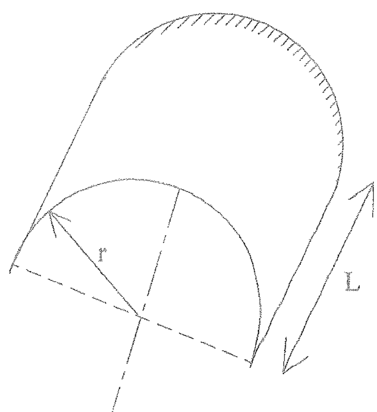

Figure Q4

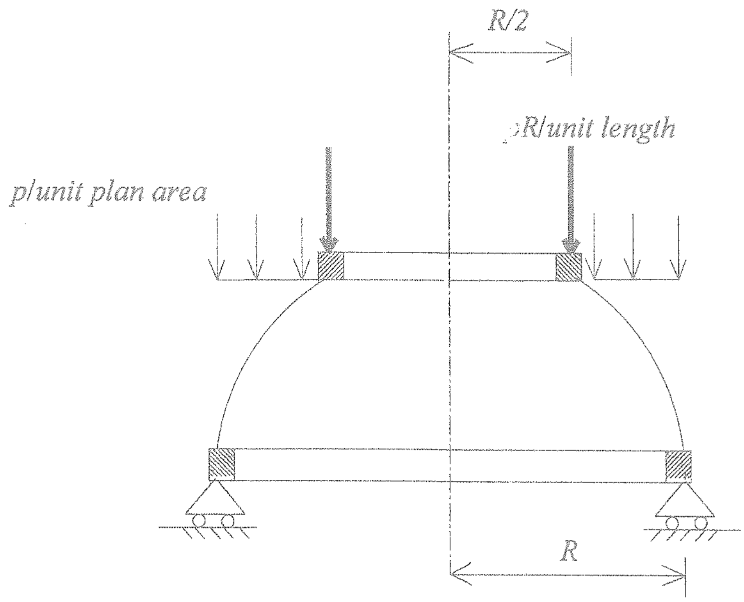

Figure Q5