UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: August 2018

Module Number: CE7203   Module Name: Computer analysis of structures

[Three Hours]

[Answer **all questions**, Marks for each question carries as indicated]

SECTION A

Q1   Fig. Q1 shows a one dimensional spring assembly. Element numbers are boxed and node numbers are circled. Elements 1 and 4 have stiffness of 2K, while elements 3 has stiffness 1K. The rest of the elements has stiffness of 3K.

    a)   Assemble the stiffness matrix of the system of springs following the node numbering given in Fig. Q1

        [4 Marks]

    b)   Calculate the deformations in node numbers 2, 3 and 4 and hence the resisting force at fixed end (i.e. Node 1)

        [3 Marks]

    c)   Comment on the selection of the node numbering adopted and the influence of node numbering on the stiffness matrix and the final outcome of the Finite element analysis.
(You may use examples of other finite element formulations to support your explanations)

        [2 Marks]

    d)   Propose an alternative means other than the assembly of stiffness matrix to compute the deformation at node numbers 2 and 3 and show that the answers obtained in this way is identical to nodal deformations found in Q1(b).

        [1 Mark]

Q2   Fig. Q2 shows an end loaded beam ABCD fixed at D pinned at B. Beam is loaded at A (free end) and C (middle of BD). Magnitudes of the two point loads at A and C are 5 kN and 10 kN, respectively. Beam ABCDE has uniform cross section and identical Young's modulus. Assume that the EI of the beam section is 1.3×10^4 kNm^2

    a)   Using a suitable element arrangement to calculate the downward deformation of the beam at A and C, assemble the global stiffness matrix of the beam ABCD.
(Use unit deformation and unit rotation at the two nodal ends of the beam element to create the stiffness matrix of the beam element. Ignore the axial effect in the formulation of the beam element matrix.)

        [4 Marks]

    b)   Based on the formulation above and the appropriate use of boundary conditions, calculate the downward deformation of the beam at points A and C.

        [2 Marks]

c)   Based on the calculations above complete the bending moment and shear force diagram of the beam ABCD.

[1 Mark]

d)   Fig. Q2(d) shows the same beam ABCD with two uniformly distributed loads applied in the beam sections AB and CD. Propose a method to analyze the beam with uniformly distributed load and explain the accuracy of your calculation based on your element selection and loading strategy.

[2 Marks]

e)   Explain the strong and weak form solution in relation to formulation of stiffness matrix of finite elements and the advantage of weak form solution in relation to complex finite element shapes and types.

[1 Mark]

Q3.   Fig. Q3 shows an idealized nineteenth century railway bridge modeled to find the structural capacity. After the investigation, it is found that all the sections of the truss elements have identical section properties and stiffness characteristics. For simplicity all the dimensions of the bar element and their section properties are represented using symbolic notations.

a)   After further investigation on the structure, its connectivity and loading suggested that the structure is symmetrical and can be represented using half the structure as shown in Fig. Q3(a). Comment the selection of half the structure to analyze the full bridge in relation to calculation of member forces, deformation at nodal points and reactions.

[2 Marks]

b)   Write down the independent element stiffness matrix and assemble the global stiffness matrix in accordance with the Fig. Q3(a), the symmetrical structure.

[3 Marks]

c)   Find the transverse (down ward) displacement of the nodal points 1 and 2

[3 Marks]

d)   Determine the member forces of all the truss element in Fig Q3(a)

[2 Marks]

Q4 A plane pin-jointed truss shown in Fig. Q4 is free to move horizontally on rollers at node 3 but it is not allowed to move vertically. It may also be assumed that the truss is firmly pinned at node 1 and the truss is loaded at free standing node 2 with 4 MN and 3 MN horizontal and vertical loads respectively. The material and geometrical properties of its members are as given below.

Member	Area (A)	Elastic modulus (E)
1-2	2A	E
1-3	A	3E
2-3	3A	2E

The element stiffness matrix [k] for a bar element in local coordinate system is given by

$$k = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ with the symbols having their usual meanings.

Determine following parameters using matrix stiffness method.

a) The horizontal and vertical displacement at node 2   [5 Marks]

b) The support reactions at nodes 1 and 3   [5 Marks]

c) The force in member 1-3   [5 Marks]

Q5. A steel frame structure shown in Fig. Q5(a) is subjected to a point load P at the mid span of beam BC. The beam is rigidly fixed to a column at one end and the other end is rested on a roller support. The base of the column is encased to a concrete pier foundation. The second moment of area (I) for the beam and the column are as given in Fig. Q5(a). Fixed end moment for a centrally loaded beam is as indicated in Fig. Q1(b). The flexibility matrix for a beam element with clockwise moment is given by $f = \frac{L}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Analyze the frame using matrix flexibility method and determine the following parameters.

a) Reaction forces at the supports A and C   [8 Marks]
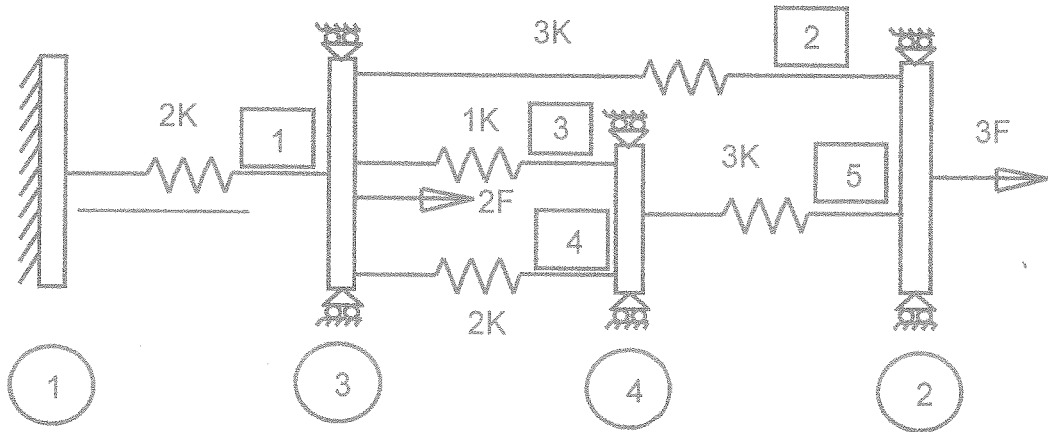
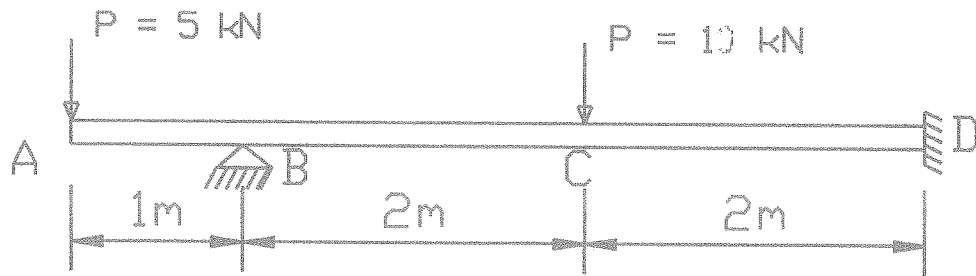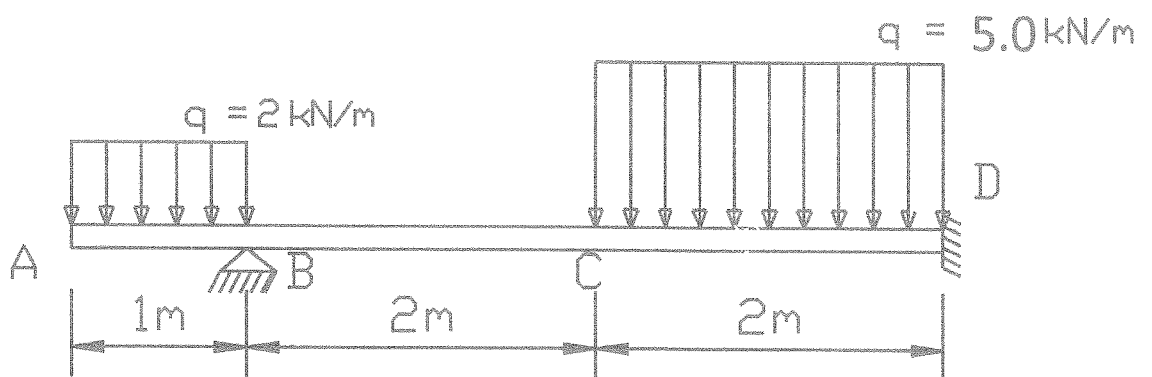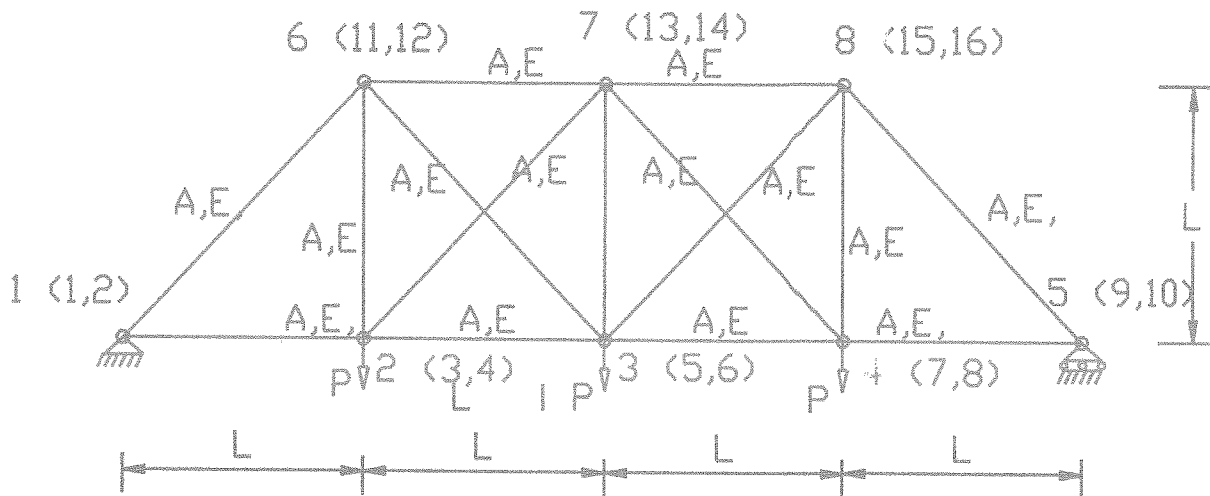b) Bending moment at joint B   [7 Marks]
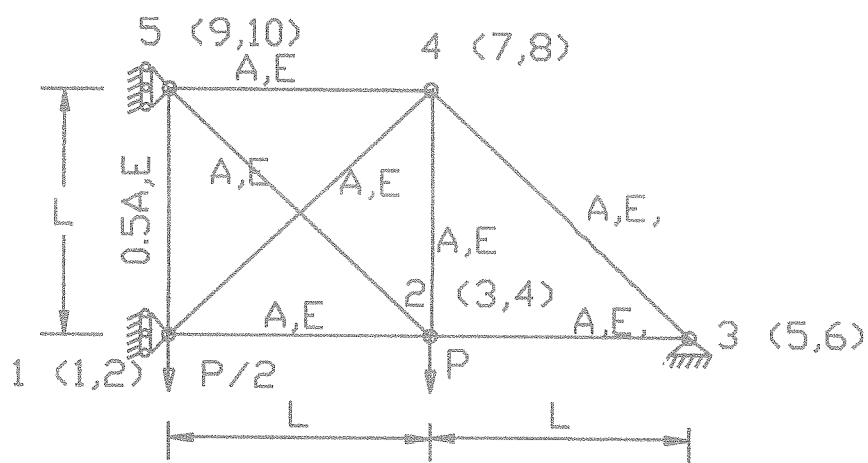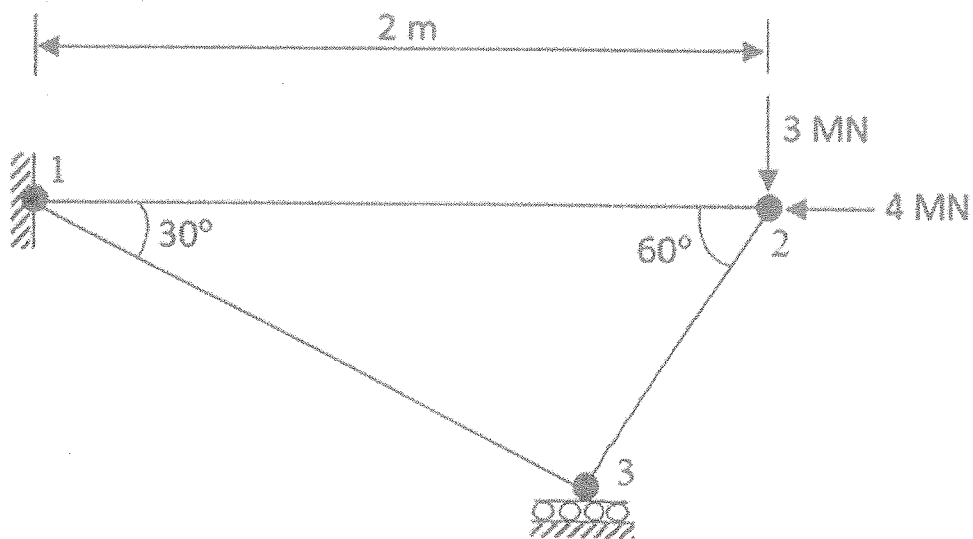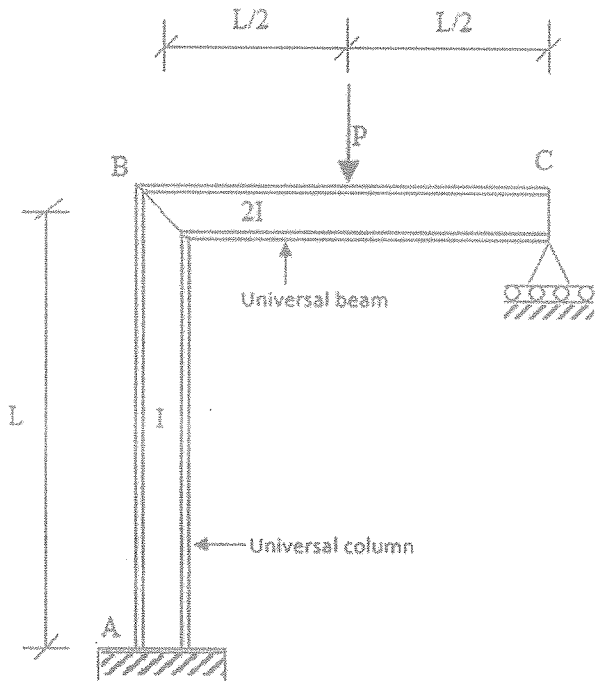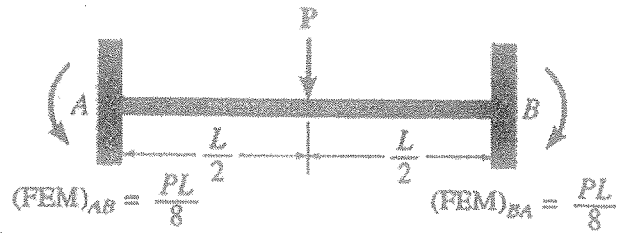
Fig. Q1

Fig. Q2

Fig. Q2 (d)

Fig. Q3

Fig. Q3 (a)

Fig. Q4

Fig. Q5 (a)

Fig. Q5(b)

$$(\text{FEM})_{AB} = \frac{PL}{8} \qquad (\text{FEM})_{BA} = \frac{PL}{8}$$