UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 2 Examination in Engineering: December 2018

Module Number: CE2302 **Module Name: Mechanics of Materials**

[Three Hours]

[Answer **all questions**, each question carries 12 marks]

All notations have their usual meanings

Q1. A tall signboard is supported by two vertical columns made of thin-walled, tapered circular tubes as shown in Fig. Q1 (a). For the purposes of analysis, each vertical column can be represented as a cantilever beam AB as shown in Fig. Q1 (b) with span L=8.0 m subjected to a point load P=2.4 kN at the free end. The column tubes have constant thickness t=10.0 mm and average diameters d_A= 90 mm and d_B=270 mm at ends A and B, respectively (see Fig. Q1(c)). Because the thickness is small compared to the diameters, the second moment of area at any cross section shall be obtained from the formula $I=[\pi d^3 t]/8$, where d=average diameter and t=wall thickness.

a) At what distance from the free end, does the maximum bending stress occur?

[6.0 Marks]

b) Corresponding to the distance found in Part (a), find the maximum bending stress in the cross section

[3.0 Marks]

c) Determine the maximum bending stress in the cross section at the support B

[3.0 Marks]

Q2. The double-web girder is constructed from two plywood sheets that are connected to wood members at the top and bottom using steel bolts as shown in Fig. Q2 (a) and (b). The girder spans 2.4 m and it is subjected to a point load, P at the mid-span. For the wood, the allowable bending stress is σ_{allow}=55 MPa and the allowable shear stress is τ_{allow}=20 MPa. The fasteners are spaced s=150 mm and each bolt can support 2.5 kN in single shear. Determine the maximum load P that can be applied at the beam mid-span without exceeding the following design criteria.

a) The allowable bending stress in the wood.

[3.0 Marks]

b) The allowable shear stress in the wood.

[4.0 Marks]

c) The allowable shear capacity of bolts.

[4.0 Marks]

d) Among the three design criteria above, identify the controlling criteria for the load P

[1.0 Mark]

Note: Neglect bolts area in determining cross sectional properties.

Q3. The stresses acting on an element at a point in the web of a wide-flange beam are found to be 75 MPa compression in the horizontal direction and 20 MPa compression in the vertical direction as shown in Fig Q3(a) with the shear stresses of magnitude 28 MPa act in the directions as shown.

a) Draw Mohr's circle to represent the normal and the shear stresses at any plane of the element at the given stress point.

[3.0 Marks]

b) Using the Mohr's circle drawn in Part (a), determine the stress components on the rotated element shown in Fig. Q3(b) and mark them accurately on the stress block based on the sign convention for Mohr's circle.

[3.0 Marks]

c) Determine principal stresses at the point and mark them on a stress block showing the angle of rotation accurately.

[3.0 Marks]

d) Determine the maximum in-plane and absolute shear stresses at the point

[3.0 Marks]

Q4. a) State two theorems associated with Moment Area Method of slope and deflection for an elastic beam.

[3.0 Marks]

b) A horizontal beam BC is freely supported at its ends on rollers placed at the free ends of two cantilever beams AB and CD as shown in Fig. Q4. The flexural rigidity (EI) of the cantilevers is twice that of the beam. The beam is loaded at its midspan by a point load of magnitude 2P. Assuming that the condition of equilibrium is static, determine following quantities using moment area method.

i) The slope and the deflection of a cantilever beam at free end B or C

[6.0 Marks]

ii) The deflection of the beam BC at mid-span E relative to a support A or D

[4.0 Marks]

iii) The slope of the beam BC at the support B

[2.0 Marks]

Q5. A steel shaft with the total length L= 4.0 m is encased for a half of its length (i.e. L/2 = 2 m) by a brass sleeve that is firmly bonded to the steel as shown in Fig. Q5. The outer diameters of the shaft and the sleeve are $d_1 = 70$ mm and $d_2 = 90$ mm, respectively and the shear modulus of steel and brass are $G_s = 80$ GPa and $G_b = 40$ GPa, respectively. Determine the allowable torques that can be applied to the shaft ends to satisfy the following conditions. Neglect the taper at point B.

a) The angle of twist, θ between the shaft ends is limited to 8.0°.

[5.0 Marks]

b) The shear stress in the brass sleeve is limited to 70 MPa.

[3.0 Marks]

c) The shear stress in the steel shaft is limited to 110 MPa.

[3.0 Marks]

d) All the three conditions stated in a), b), and c) to be satisfied.

[1.0 Mark]

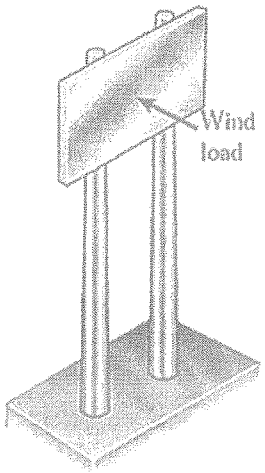

Fig. Q1 (a)

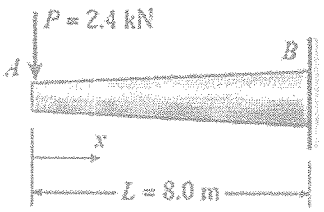

Fig. Q1 (b)

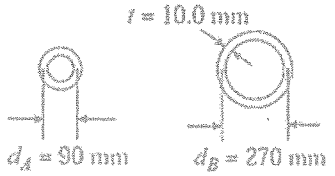

Fig. Q1 (c): Cross sections at A and B

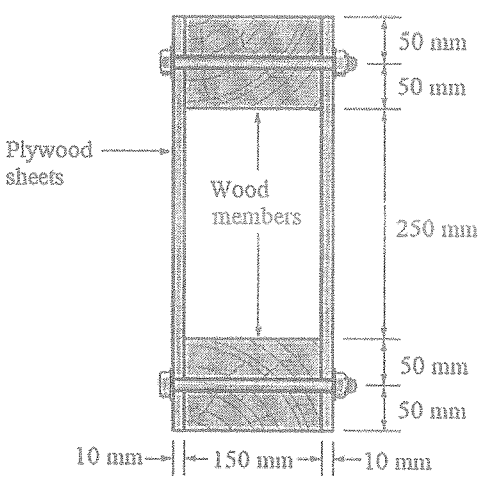

Fig. Q2 (a)

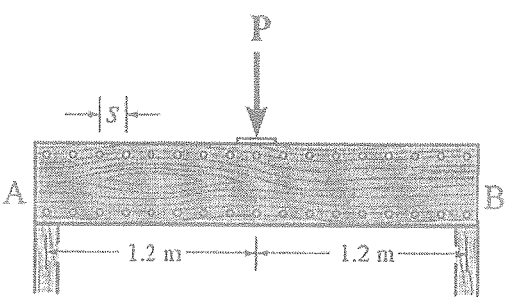

Fig. Q2 (b)

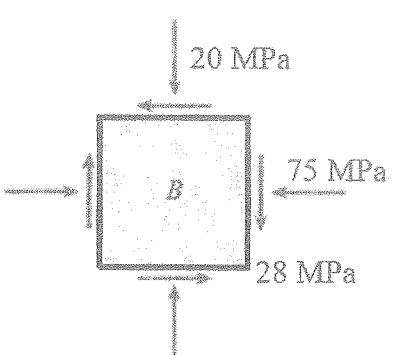

Fig. Q3 (a)

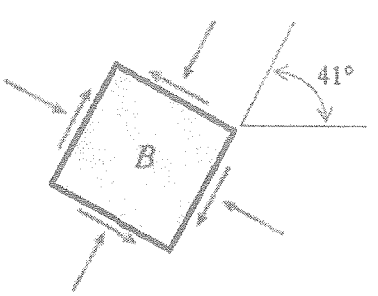

Fig. Q3 (b)

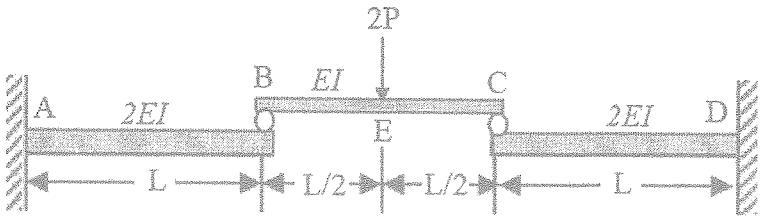
2P
A
2EI
B
EI
C
E
2EI
D
L
L/2
L/2
L

Fig. Q4.

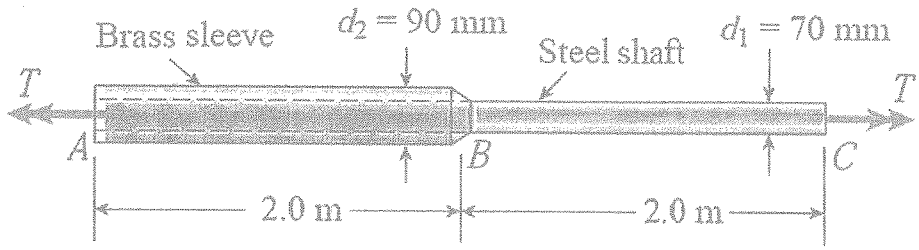
Brass sleeve
$d_2 = 90$ mm
Steel shaft
$d_1 = 70$ mm
T
T
A
B
C
2.0 m
2.0 m

Fig. Q5.