UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: December 2018

Module Number: EE6302 **Module Name: Control system Design**

[Three Hours]

[Answer **all questions**, each question carries 15 marks]

All notations have their usual meanings. Formulas you may require is given in page 4 and Laplace transform table is given in Table No 1, page 7.

Q1. a) i) Sketch a suitable time response to illustrate the *rise time, overshoot* and *settling time*.

ii) Give expressions for the time domain specifications given in part a).i) for the system given in Figure Q1(a).

iii) Sketch the allowable region in the S-Plane for the locations of poles of a second order unit step response whose time domain specifications are to be kept as; *rise time* $\leq a$ s, *overshoot* $\% \leq b\%$ and *settling time (1% criteria)*$\leq c$ s, where a, b and c are real numbers.

[5.5 Marks]

b) You are asked to design a simple speed control system of a DC motor as shown in Figure Q1 (b). The motor is running at no-load with negligible load torque. For a unit impulse armature voltage of the DC motor $v_a(t)$, the angular speed $\omega_m(t)$ is expressed as

$$\omega_m(t) = 10[e^{-2t} - e^{-10t}]$$

i) Obtain the transfer function of the DC motor and hence, find the equivalent circuit parameters of the DC motor R_a, L_a, J, and b shown in Figure Q1 (b).

ii) Calculate the value of the PE actuator constant A at which the DC motor speed control system undergoes critically damped stage.

iii) Determine the factor by which A should be multiplied to reduce the damping ratio(ξ) from critical stage to 0.6.

iv) With this motor speed control system, it is required to track the speed reference of 200 $rads^{-1}$. Find the rated speed of the DC motor, in order that the motor speed control system be a stable system with $\xi = 0.6$.

v) Hence, discuss the reference tracking capability of this DC motor speed control system.

[9.5 Marks]

Directions : Those who have not answered part b)i), can find the answers for the rest of parts in terms of R_a, L_a, J, and b.

Q2. a) i) Write the general form of the matrix equations so that a system is represented in a state-space model. Name the matrices in your matrix equations.

ii) Derive the transfer function of the system using the matrix equations written in part a) i).

iii) State the Routh's necessary and sufficient conditions to have a stable system.

[4.0 Marks]

b) It is required to evaluate the stability of a plant G(s), which consists of two subsystems G1 and G2, connected in series. The block diagram of $G_1(s)$ is shown in Figure Q2 (b1) and Figure Q2 (b2) illustrates the locations of the poles and zeroes of $G_2(s)$ in the s-plane.

i) If the state vector of the plant can be denoted as $[x_1 \quad x_2 \quad \cdots \quad x_n]$ and the block diagram has 6, 5, -4 and K for k_1, k_2, k_3 and k_4, respectively, formulate the state-space model of the subsystem $G_1(s)$ shown in Figure Q2 (b1).

ii) Hence, obtain the transfer function of $G_1(s)$.

iii) Find the range of K for which the plant G(s) is stable.

[6.0 Marks]

c) i) What are the advantages of feedback control when compared with open-loop control?

ii) Figure Q2 (c1) and Q2 (c2) illustrate the block diagrams of an open-loop and a closed loop control system of a plant. The transfer function of the plant is

$$G(s) = \frac{A}{Ts+1}$$

The controller is a proportional controller with gain K.

Derive expressions for steady state values of the plant output for the open loop control system and the closed loop control system when $W(s) = \frac{w}{s}$ and $R(s) = \frac{r}{s}$. Here, w and r are constants.

Using the expressions you have derived, comment on the impact of disturbance on the response of the open and closed loop control systems.

[5.0 marks]

Q3. a) i) State the definition of the root locus.

ii) Explain how you find the system gain to operate the closed loop transient response at a given percent overshoot using root locus.

[4.0 marks]

b) Consider the unity feedback system shown in Figure Q3 where

$$G(s) = \frac{K}{(s+10)(s^2+4s+5)}$$

i) Plot the root locus for this system.

ii) Find the range of gain K where the closed loop system is stable.

iii) Find the value of gain K that yields 15% overshoot in the closed loop step response.

iv) Find the closed loop poles of the system for the gain you have found in part b)iii).

v) Evaluate the accuracy of the second order approximation of the closed loop transfer function for the gain you have found in part b)iii).

vi) Estimate the settling time and the peak time of the closed loop system operating at 15% overshoot.

vii) Calculate the steady state error of the gain adjusted system for a unit step input.

viii) Design a suitable compensator for the gain adjusted system with K= 56.89 to improve the steady state error by a factor of 10.

[11.0 marks]

Q4. a) i) Define the terms phase margin and gain margin associated with Bode plots.

ii) Explain how you evaluate the stability of a closed-loop system using the frequency response of an open-loop system.

[4.0 marks]

b) Consider the unity feedback system shown in Figure Q4 where

$$G(s) = \frac{K}{(s+2)(s+3)(s+12)}$$

i) Find the steady state error of the closed-loop system for a unit step input when $K = 1$ and $G_c(s) = 1$.

ii) Find the required gain to improve the steady state error of the closed-loop system by a factor of 20.

iii) Sketch the bode magnitude and bode phase responses of $G(s)$ for the gain K you have found in part b).ii).

iv) Evaluate the stability of the closed loop system for the gain you have found in part b).ii) using the frequency response of the open-loop system.

v) Using the frequency response, find the range of gain K where the closed-loop system is stable.

vi) Design a lag compensator for the gain adjusted system with $K = 658.2$ to improve the percent overshoot to 15%. Use the frequency response design technique.

[11.0 marks]

Formulas you may require:

$$\phi_M = \tan^{-1}\left(\frac{2\xi}{\sqrt{-2\xi^2 + \sqrt{1 + 4\xi^4}}}\right)$$

$$\omega_{BW} = \frac{4}{T_s \xi}\sqrt{(1 - 2\xi^2) + \sqrt{4\xi^4 - 4\xi^2 + 2}}$$

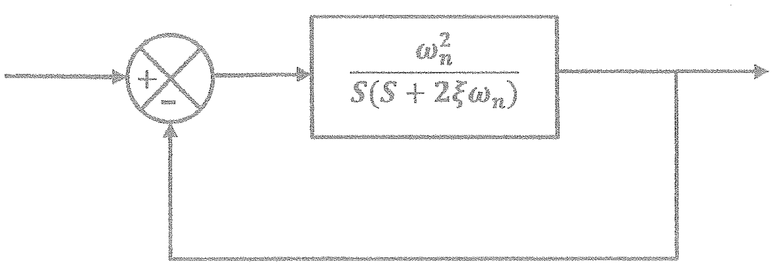

Figure Q1 (a).

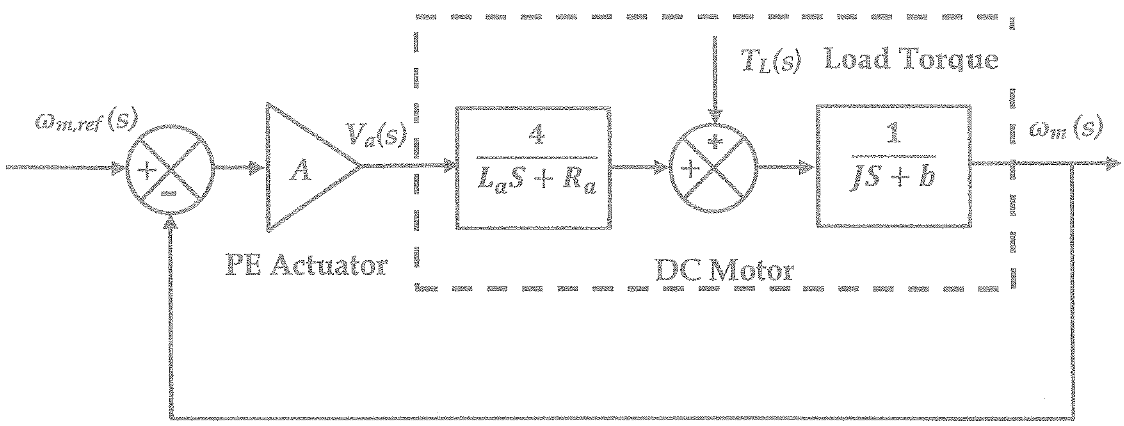

Figure Q1 (b).

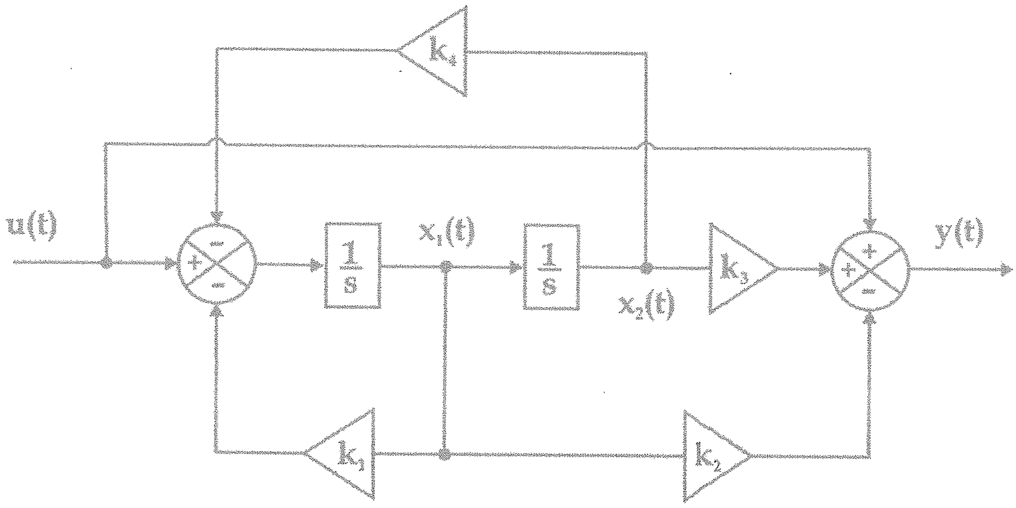

Figure Q2 (b1).

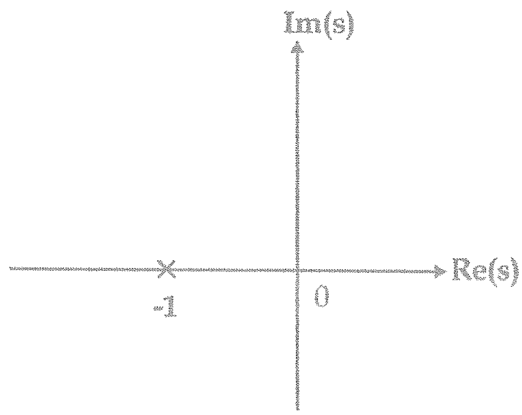

Figure Q2 (b2).

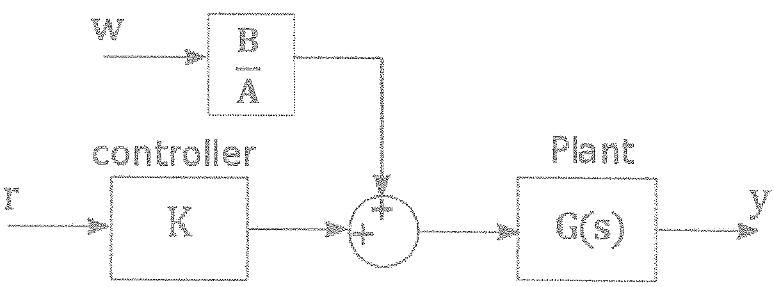

Figure Q2 (c1) Open-loop control system.

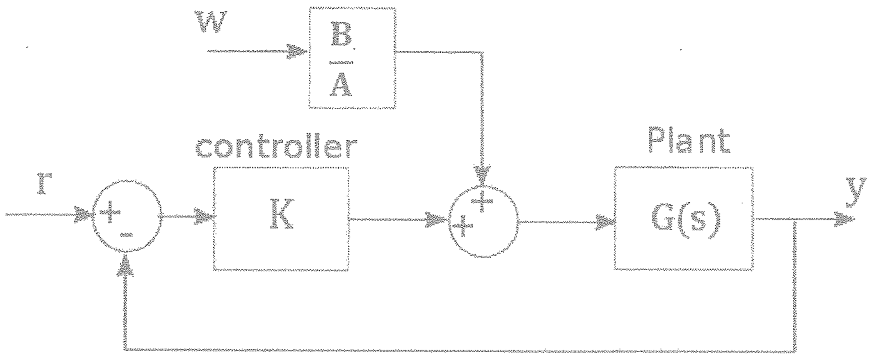

Figure Q2 (c2) Closed-loop control system.

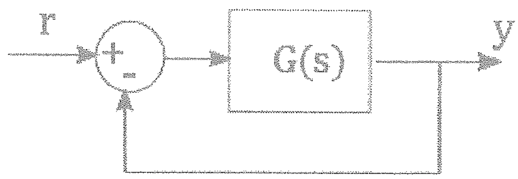

Figure Q3.

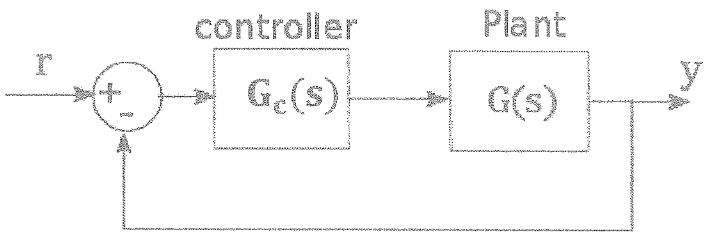

Figure Q4.

Table No 1: Laplace Transform Table

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$1/s$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\int f(t)dt$	$\frac{F(s)}{s}+\frac{1}{s}\int(f(t)dt)\|_{t=0}$