UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: December 2018

Module Number: IS6301 **Module Name: Mathematical Modelling**

[Three hours]

[Answer **all questions**, each question carries fourteen marks]

Q1.

a) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to each solution? What is the added value in each case?

[2 Marks]

b) Fluids A, B, C and D contain the substances S_1, S_2 and S_3 (concentrations in grams per liter) according to the table shown below.

	A	B	C	D
S_1	2.5	8.2	6.4	12.7
S_2	3.2	16.5	13.2	0.4
S_3	1.1	0.9	2.2	3.5

What is the concentration of S_3 in a mixture of these fluids that contains 75% (percent by volume) of fluids A and B and which contains $4gl^{-1}$ and $5gl^{-1}$ of the substances S_1 and S_2, respectively?

[6 Marks]

c) One dimensional simple harmonic motion for the system, shown in Figure Q1 can be represented by a second order differential equation as below. The equations can be obtained using *Newton's second law of motion* ($F=ma$) and *Hooke's Law* ($F=-ku$).

$$m\frac{d^2u_1}{dt^2} + 3ku_1 - ku_2 = 0$$

$$m\frac{d^2u_2}{dt^2} - 2ku_1 + 2ku_2 = 0$$

Where; m = mass, u = displacement, k = spring constant and subscripts 1 and 2 denote particles 1 and 2 respectively.

Given that k=1 and m=1, find the natural frequencies of vibration of the system.

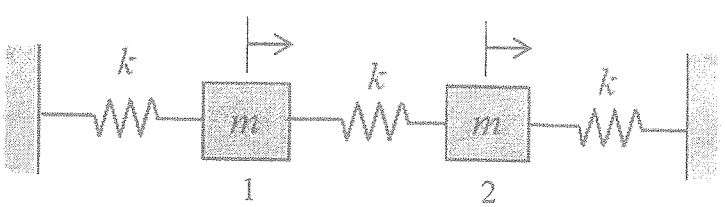

Figure Q1

[6 Marks]

Q2.

a) i.) Clearly state the 'Buckingham Pi Theorem'.

ii.) Briefly discuss what is meant by 'Pivot column' and 'Pivot row' in a simplex tableau.

[2 Marks]

b) A long structural component of a bridge has the cross section shown in Figure Q2. It is known that when a steady wind blows past this type of bluff body, vortices may develop on the downwind side that are shed in a regular fashion at some definite frequency. Since these vortices can create harmful periodic forces acting on the structure, it is important to determine the shedding frequency. For the specific structure of interest, D=0.2m, H=0.4m, and the representative wind velocity (V) is 50km/hr. Standard air can be assumed. The shedding frequency is to be determined through the use of a small-scale model that is to be tested in a water tunnel. For the model D_m=20mm and the water temperature is 20°C.

i.) Perform dimensional analysis to obtain the relevant non-dimensional groups.

ii.) Determine the model dimension (H_m), and the velocity (V_m) at which the test should be performed.

iii.) If the shedding frequency (ω) for the model is found to be 49.9Hz, what is the corresponding frequency for the prototype?

You may use, viscosity (μ_{air}) = 1.79×10^{-5} kg m^{-1} s^{-1} and density (ρ_{air}) = 1.23 kg m^{-3}, for air at standard condition. For water at 20°C, viscosity (μ_{water}) = 1.0×10^{-3} kg m^{-1} s^{-1} and density (ρ_{water}) = 998 kg m^{-3}.

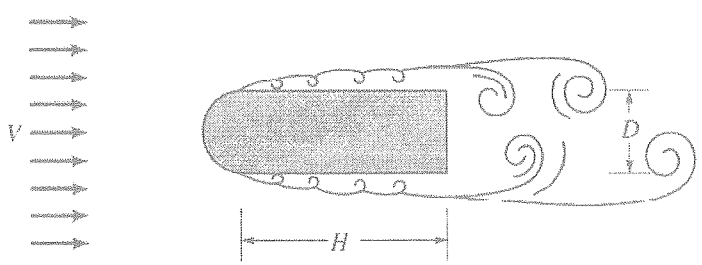

Figure Q2

[7 Marks]

c) A family has a farm of 900 hectares and they are planning to grow soy beans, pumpkin, and sweet corn in the planting season. Fertilizer costs per hectare are: Rs. 500 for soy beans, Rs. 200 for pumpkin and Rs. 100 for sweet corn. It was estimated that each hectare of soy beans will require an average of 5 hours of labor per week while tending pumpkin and sweet corn should only require about 2 hours each per week. Based on past yields and current market trends, the estimated profits are: Rs. 300,000 for each hectare of soy beans, Rs. 200,000 for each hectare of pumpkin and Rs. 100,000 for each hectare of sweet corn. The family can afford to spend no more than Rs. 300,000 on fertilizer and their workers combined are willing to work at least 2000 hours per week. Estimate the number of hectares of each crop should be planted to maximize your profit, by using Simplex method.

[5 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of 'Vogel's Approximation Model (*VAM*)' and 'Modified Distribution Method (*MODI*)' used to solve transportation problems.

[2 Marks]

c) Oranges are grown, picked, and then stored in warehouses in Bibile, Badulla and Monaragala. These warehouses supply oranges to markets in Ampara, Ratnapura, Kandy and Galle. The following table shows the shipping costs per truckload (Rs. 1000s), supply and demand.

From	To				Supply
	Ampara	Ratnapura	Kandy	Galle	
Bibile	9	14	12	17	200
Badulla	11	10	6	10	200
Monaragala	12	8	15	7	200
Demand	130	170	100	150	

i.) Set up the transportation tableau for this problem and determine the initial solution using the minimum cell cost method.

ii.) Solve the problem using Modified Distribution Method *(MODI)*.

iii.) Are there multiple optimal solutions? Explain. If so, identify them.

[8.0 Marks]

d) Suppose, in question part (c), the shipments are prohibited from Badulla to Kandy due to ongoing road construction works. What modification to the transportation tableau would you do to consider this prohibited root? Briefly explain any specific points of solving this problem with this modification. (Do not perform any calculation again)

[2.0 Marks]

Q4.

a) Briefly explain what is meant by,

i.) 'Row reductions' and 'Column reductions' performed in assignment problems.

ii.) Prohibited Assignment.

[2 Marks]

b) Give examples (2 each) for maximization and minimization assignment problems occurring in real world with related to engineering/management.

[2 Marks]

c) A local Airline company must staff the daily flights between New York and Chicago as shown in the table below. Each of the Airline's crew lives in either New York or Chicago. Each day a crew must fly one New York-Chicago and one Chicago-New York flight with at least 1-hour downtime between flights. The Airline wants to schedule the crews to minimize the total downtime. Setup an assignment problem that can be used to accomplish this goal. (*Hint:* Let $x_{ij} = 1$ if the crew that flies flight i also flies flight j, and $x_{ij} = 0$ otherwise. If $x_{ij} = 1$, then a cost C_{ij} is incurred, corresponding to the downtime associated with a crew flying flight i and flight j). According to the flight schedule, it is noticeable that some assignments are not feasible.

Flight	Leave Chicago	Arrive New York	Flight	Leave Chicago	Arrive New York
1	6 a.m.	10 a.m.	1	7 a.m.	9 a.m.
2	9 a.m.	1 p.m.	2	8 a.m.	10 a.m.
3	12 noon	4 p.m.	3	10 a.m.	12 noon
4	3 p.m.	7 p.m.	4	12 noon	2 p.m.
5	5 p.m.	9 p.m.	5	2 p.m.	4 p.m.
6	7 p.m.	11 p.m.	6	4 p.m.	6 p.m.
7	8 p.m.	12 midnight	7	6 p.m.	8 p.m.

Find the flight assignments that minimize the total downtime using 'Hungarian' method. Calculate the total minimum downtime. Assume that at the end of the day, each crew must be in its home city.

[10 Marks]

Q5.

a) Find the equilibrium point, if it exists, for the following dynamical system. State whether the equilibrium point stable or unstable.

$$u(n) = 2u(n-1) + v(n-1) + 3$$

$$v(n) = 4u(n-1) - v(n-1) + 6$$

[2.0 Marks]

b) Assume that,

- there are 2 chemicals in the body, U and V
- the body filters out 10% of U and 15% of V each day through the kidneys
- liver enzymes metabolize 30% of U into V and 25% of V into U each day.

In addition, assume that, each day x mg of V is taken in but no U is taken in.

i.) What should x be so that the equilibrium value for V is 200mg?

ii.) What will be the resulting equilibrium value for U given the prescribed dosage of V found in part (i)?

[6.0 Marks]

c) To buy a boat in 2 years, Mr. Silva's family plans to save Rs. 20,000 a month in an account that pays 12% interest, compounded monthly.

i.) Find the total amount of the payments.

ii.) Find the value of the account in 2 years.

iii.)Find the single deposit in that account that would give the same future value.

[6.0 Marks]