UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: July 2017

Module Number: ME 3303 **Module Name: Modelling of Dynamic Systems**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

A partial table of Laplace transformation pairs is given in page 7. You may make additional assumptions where necessary, but clearly state them in your answers. Symbols stated herein denote standard parameters.

Q1 a) Use Laplace Transform to find the solution for the following differential equation which has non-constant coefficients.

$$t\ddot{y}(t) - t\dot{y}(t) + y(t) = 2; \quad y(0) = 2,\ \dot{y}(0) = -4$$

[4.0 Marks]

b) Use Convolution Integral Theorem in Laplace Transform to find the solutions for the following ordinary differential equation.

$$\ddot{y}(t) + y(t) = \tan(t); \quad y(0) = 1,\ \dot{y}(0) = 2$$

Hint: Uniqe solution to the initial value problem $a\ddot{y}(t) + b\dot{y}(t) + cy(t) = g(t)$ is given by *y(t)= u(t)+(h*g) (t)* where *u(t)* is the solution of the homogenous equation $a\ddot{u}(t) + b\dot{u}(t) + cu(t) = 0$ and *h(t)* has the Laplace Transform $H(s) = \frac{1}{as^2 + bs + c}$. The initial values remain same for both *u(t)* and *y(t)*. The convolution integral of *h(t)* and *g(t)* is noted as *(h*g) (t)*.

[5.0 Marks]

c) Figure Q1(c) illustrates the first order rise for a unit step input and it is described by $y(t) = b(1 - e^{-at})$ where a and b are constants. The time constant τ is defined as the time at which the gradient at t=0 intersects the steady stage level. Show that the response reaches approximately 63% of its steady state level after time constant.

[3.0 Marks]

Q2 a) Briefly explain the importance of modeling and simulation when developing a mechatronic product?

[1.0 Mark]

b) Runge-Kutta, Forward Euler, Backward Euler and Trapez method are some of the commonly used numerical techniques in time domain simulation. Briefly explain the reasons for not selecting

i) a too smaller value or

ii) a too larger value

for the 'time-step' in above time based iteration techniques.

[3.0 Marks]

Q2 is continued to next page…

c) Figure Q2(c) shows two bodies that can only rotate. The first body is connected to the ground via a rotational spring and a rotational damper. The second body is connected to the first body via a rotational spring and a rotational damper. The following data is given: $J_1=J_2= 2\ kgm^2$, $k_{\theta 1}= k_{\theta 2}=1800\ Nm/rad$ and $b_{\theta 1}= b_{\theta 2}=8\ Nms/rad$. The rotational spring between ground and the first body is undeformed when $\theta_1=0\ rad$ and the second spring is undeformed when $\theta_1= \theta_2$. Four different situations are to be investigated. They are characterized by the external applied moment $M(t)=M_0 \sin(\omega_p t)$.

Situation	Size of applied Moment, M_0 [Nm]	Frequency, ω_p rad/s
(a)	0	N/A
(b)	75	18.5
(c)	75	28.5
(d)	75	38.5

It is assumed same initial conditions for all four situations: $\theta_1(t=0)=25^\circ$, $\theta_2(t=0)=-25^\circ$ and $\dot{\theta}_1(t=0)=\dot{\theta}_2(t=0)=0\ rad/s$. Develop a Matlab program to simulate the situation (a) of the system for a total time of T=4s, using following guidelines.

```
%Time-Domain Simulation of Rotating Spring-Mass-Damper System
close all;
clear;
%Given Data

.............................  %(Add Lines Here)

%Initial Conditions;
Time=0;

.............................  %(Add Lines Here)

counter=1;
EndTime=4;
%Start Time Integration
while Time<EndTime
   %Store Values for Plotting

   .............................  %(Add Lines Here)

   %Time Integration Using Forward Euler Method

   .............................  %(Add Lines Here)

   %Updating System Equations
   Theta1DotDot=(k*(Theta2-Theta1)+b*(Theta2Dot-Theta1Dot)-k*Theta1-
   b*Theta1Dot)/j;
   Theta2DotDot=(M0*sin(wp*Time)-k*(Theta2-Theta1)-b*(Theta2Dot-
   Theta1Dot))/j;
   Time=Time+StepTime;
   counter=counter+1;
end;
%Draw Plots

.............................  %(Add Lines Here)

%End of the Program
```

[7.0 Marks]

Q2 is continued to next page…

d) Outline a procedure to repeat the simulations for remaining situations (b), (c), (d) using the same Matlab code developed in situation (a).

[1.0 Mark]

Q3 a) A plant is described by the following differential equation.

$$\ddot{y}(t)+4\dot{y}(t)+3y(t)=\dot{u}(t)+5u(t)$$

Determine the open loop transfer function $G_{OL}(s) = Y(s)/U(s)$ of the plant.

[2.0 Marks]

b) Draw the closed loop control system for the plant using single feedback gain K and derive the closed loop transfer function.

[2.0 Marks]

c) Determine the value(s) of K for critical damping of the response.

[3.0 Marks]

d) Calculate DC gain(s) of the critically damped closed loop plant.

[1.0 Mark]

e) Show that an additional gain of 1+(5/3)K is needed to maintain zero steady state error.

[2.0 Marks]

f) Can you use critically damped controllers in practical mechatronic applications? Justify your answer with examples.

[2.0 Marks]

Q4 a) A simplified DC servomotor model is shown in Figure Q4(a). The input is the armature voltage $v(t)$ and the output is the angular displacement of the motor shaft $\theta(t)$. The constant parameters are armature circuit inductance and resistance L and R, respectively, and motor shaft polar inertia and rotational viscous damping coefficient J and b, respectively. The intermediate variables are armature current $i(t)$, motor torque $\tau(t)$, and motor shaft angular velocity $\omega(t)=\dot{\theta}(t)$. In this simplified model, back emf voltage, gear ratio and load inertia are neglected. The dynamic model of this system can be derived by combining circuit model, electromechanical coupling, and rotational mechanical model. Motor torque is proportional to the armature current, thus the electromechanical coupling equation is given by $\tau(t)=K_T\, i(t)$, where K_T is the motor torque constant. For the rotational mechanical model, Euler's rotational law results in the second order differential equation relating the motor shaft angle $\theta(t)$ to the motor torque $\tau(t)$; $\tau(t)=b\dot{\theta}(t)+J\ddot{\theta}(t)$.

i) Obtain dynamic model of the overall system using Laplace Transform to describe the motor shaft angular position; $\theta(s)$ (output variable), as a function of armature voltage; $V(s)$ (input variable).

ii) Use inverse Laplace transform to yield a third-order linear time-invariant ordinary differential equation which describe the dynamic model in part i)

iii) Obtain the state space model of the system (state equation, output equation and A, B, C, D matrices) with usual notations.

Q4 is continued to next page…

iv) Draw a Matlab Simulink diagram to represent the block diagram of the above state space system.

[8.0 Marks]

b) The servomotor described in part a), can be numerically represented by following state and output equations.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u],$$

$$[\theta] = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Figure Q4(b) shows a state feedback regulator which is to be implemented for the servomotor system. Design a full-state-feedback regulator fulfilling pole placement requirements at $-2 \pm j4$, -10.

[4.0 Marks]

Q5 a) Lyapunov first method (Indirect method) determines the stability nature of the equilibrium state (at the origin) of a nonlinear autonomous dynamic system. The method generalizes the concept of energy; V for a conservative system in mechanics, where a well-known result states that an equilibrium point is stable if the energy is minimum. Thus V is a positive function which has $\dot{V}$ negative in the neighborhood of a stable equilibrium point. Apply Lyapunov first method to determine the stability of the simple pendulum shown in Figure Q5 (a). Neglect any friction effects.

[4.0 Marks]

b) The simplified dynamics of a magnetically suspended steel ball are given by

$$m\ddot{y} = mg - c\frac{u^2}{y^2};$$

where the input u represents the current supplied to the electromagnet, y is the vertical position of the ball, which may be measured by a position sensor, g is gravitational acceleration, m is the mass of the ball, and c is a positive constant such that the force on the ball due to the electromagnet is $c\frac{u^2}{y^2}$. Assume a normalization such that $m = g = c = 1$.

i) Using the states $x_1 = y$ and $x_2 = \dot{y}$; write down a nonlinear state space description of this system.

ii) What equilibrium control input u_e must be applied to suspend (at zero velocity) the ball at $y = 1\ m$?

iii) Write Lyapunov's linearized state space equations for state and input variables in a close neighborhood of the equilibrium point obtained in the part ii).

iv) What can you conclude about the stability of the nonlinear system close to the equilibrium point x_e?

[8.0 Marks]

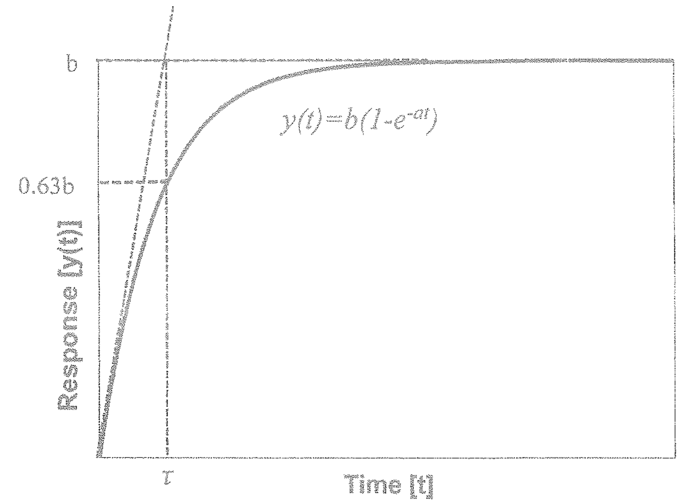

Figure Q1(c): First Order Rise

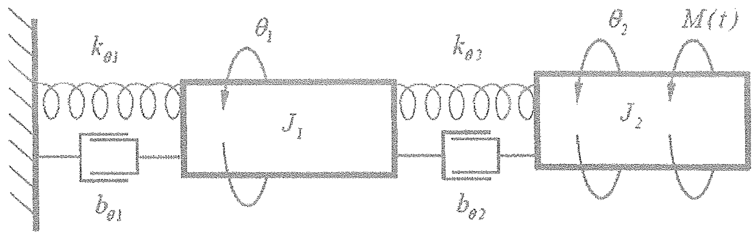

Figure Q2(c): System with Rotating Spring-Damper Inertias

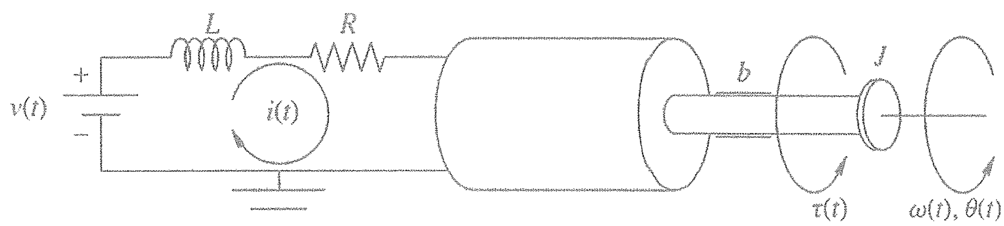

Figure Q4(a): Simplified DC Servomotor Model

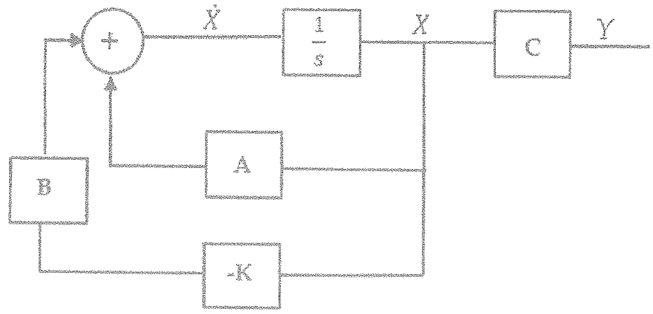

Figure Q4(b): Full State Feedback Regulator

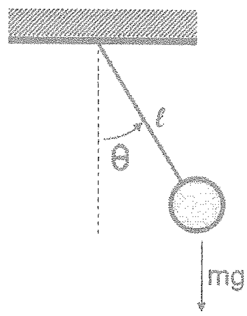

Figure Q5(a): Simple Pendulum

Table of Laplace transform pairs

$f(t)$	$F(s)$
step	$\frac{1}{s}$
ramp, t	$\frac{1}{s^2}$
impulse	1
dirac delta function, $\delta(t-c); c \geq 0$	e^{-cs}
$u(t-a)$	$\frac{e^{-as}}{s}$
$u(t-a)\ g(t-a)$	$e^{-as}\,G(s)$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin(\omega_0\sqrt{1-\zeta^2}\,t)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
$1-\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin(\omega_0\sqrt{1-\zeta^2}\,t+\phi),\quad Where,\ \phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$
$\frac{d(f(t))}{dt}$	$sF(s)-f(0)$
$\frac{d^2(f(t))}{dt^2}$	$s^2F(s)-sf(0)-\dot{f}(0)$
$\int f(t)dt$	$\frac{1}{s}F(s)+\frac{1}{s}\left[\int f(t)dt\right]_{t=0}$
$f(t-\alpha)$	$e^{-\alpha s}F(s)\,with\,f(t-\alpha)=0,\ t\leq\alpha$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$
$t^n f(t);\quad n=1,2,3$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$f(t/a)$	$aF(as)$
Convolution Intergral; $(f_1 * f_2)(t)=\int_0^t f_1(t-\tau)f_2(\tau)d\tau$	$F_1(s)F_2(s)$