UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examinaton in Engineering: August 2017

Module Number: ME5301 Module Name: Computer Aided Design

[Three Hours]

[Answer all questions. All questions carry equal marks]

Q1. a) Determine the algebraic coefficients of a parametric quadratic curve which connects the points (0, 1, 0), (1, 2, 4) and (2, 4, 5). Assume that the three given points are parameterised at u = 0, 0.5 and 1 respectively. Also find $\frac{dy}{dx}$ and $\frac{dz}{dx}$ at both ends of the curve.

[6.0 Marks]

b) A PC curve was derived in class for end point positions and tangent constraints, but these are not the only geometric constraints that could be used. Develop a similar PC curve for end points and curvature constraints. (i.e. develop matrix equations)

Take:
$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 6 & 2 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & -1/6 & 1/6 \\ 0 & 0 & 1/2 & 0 \\ -1 & 1 & -1/3 & -1/6 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

[6.0 Marks]

Q2. a) Discuss the advantages of B-splines over Bezier curves.

[2.0 Marks]

b) Suppose that we join two Bezier curves of degree 2, using the control point sequences (P_0, P_1, P_2) and (P_2, P_3, P_4), respectively. What conditions must be satisfied by these five points to have C^1 continuity at the join point?

[2.0 Marks]

c) i) Prove that a Bezier curve with 4 control points can be described by p = UMP where U = [u^3 u^2 u 1]

P = $[P_0\ P_1\ P_2\ P_3]$ and

$$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

[4.0 Marks]

ii) Using the above relationship p = UMP, construct the Bezier curve whose control points are described by (0,0), (50,100), (100,150) and (150,0).
(Use the graph sheet to plot the curve.)

[4.0 Marks]

Q3 a) List three modelling approaches and briefly explain one by one clearly.

[3.0 Marks]

b) Derive the DDA algorithm for a line starting from left end point to the right end point of a XY Cartesian Coordinate system and the gradient $0 \le m \le 1$ for a given number (n) of input points.

[2.0 Marks]

c) Derive the Bresenham's Algorithm for drawing a line starting from left end point to the right end point of a XY Cartesian Coordinate system and the gradient $0 \le m \le 1$ for a given number (n) of input points.

[3.0 Marks]

d) A line to be drawn from (30, 40) mm to (330, 225) mm on a display which is mapped to approximately (320X240)mm. The resolution of the screen is 640X480. Calculate five points in pixel starting from start point (30, 40) by using Bresenham's Algorithm.

[4.0 Marks]

Q4 A given XYZ Cartesian coordinate frame "O" undergoes a series of consecutive transformations as given below.

i) a transformation which corresponds to a rotation of 90° about Y-axis
ii) a transformation which corresponds to a translation of 10 units in X direction, 20 units in Y direction and 30 units in Z direction.
iii) a transformation which corresponds to a rotation of 90°about Y-axis
iv) a reflection through Y axis

a) Write down the homogeneous transformation matrices for each transformation.

[4.0 Marks]

b) Calculate the concatenated transformation matrix. Show all the steps.

[2.0 Marks]

c) Transform a given shape defined by the points U1=7i+5j+3k, U2=7i+2j+3k, U3=7i+5j+8k, U4=7i+2j+8k according to the defined working coordinate system "O" to Model coordinate system .

[4.0 Marks]

d) Draw the working coordinate system "O" on an XYZ Cartesian coordinate frame (Model coordinate frame) and draw the specified shape.

[2.0 Marks]

Q5 a) State Castigliano's first theorem for elastic systems in equilibrium.

[1.0 Marks]

b) Using Castigliano's first theorem derive element stiffness matrix for the system shown in Figure Q5(a) and solve for the displacements and the reaction force at node 1 if,

$K_1 = 5$ N/mm , $K_2 = 8$ N/mm, $K_3 = 3$ N/mm

$F_2 = -30$N , $F_3 = 0$, $F_4 = 50$N

[3.0 Marks]

c) The figure Q5(b) shows local nodal displacements and global nodal displacements of a bar element which has an orientation of θ. Derive the transformation matrix [R] which converts global nodal displacements to local nodal displacements.

[2.0 Marks]

d) The plane truss shown in Figure Q5(c) is composed of members having a square 15 mm × 15 mm cross section and modulus of elasticity $E = 69$ GPa.

i) Assemble the global stiffness matrix.

ii) Compute the nodal displacements in the global coordinate system for the loads shown in Figure Q5(c).

iii) Compute the axial stress in each element.

The element stiffness matrix in global coordinate frame for a bar element with an orientation of θ is given by ,

$$[k^{(e)}] = k_e \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Where c=cos θ and s=sin θ

[6.0 Marks]

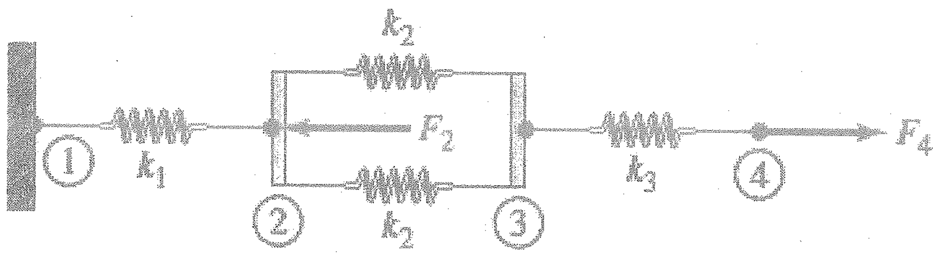

Figure Q5 (a)

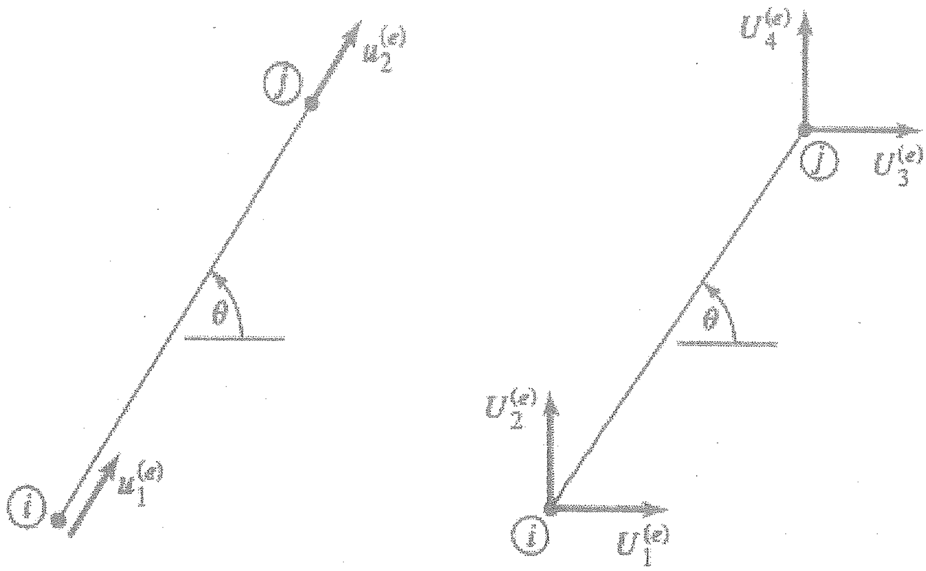

Figure Q5 (b)

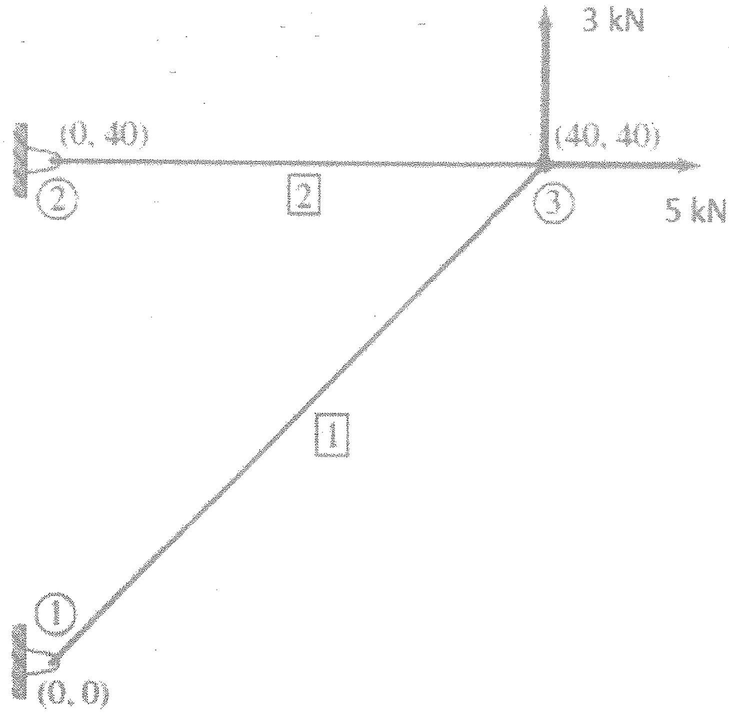

Figure Q5 (c)