# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 5 Examination in Engineering: July 2017

**Module Number: IS5301** **Module Name: Numerical Methods**

**[Three hours]**

[Answer **all questions**, each question carries 14 marks]

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Q1. a) Write down three examples where the numerical methods offer a net benefit to the engineer. [1.5 Marks]

b) i) Briefly explain the bisection method.

[2.0 Marks]

ii) To find the inverse of the number $a$ $(a \in R)$, one can use the equation

$$f(x) = a - \frac{1}{x} = 0$$

where $x$ is the inverse of $a$.

Use the bisection method of finding roots of equations to find the inverse of $a = 2.5$. Compute three iterations to estimate the root of the above equation, by starting with the interval [0,1].

[5.0 Marks]

iii) Find the absolute relative approximate error at the end of each iteration in above (ii). [3.0 Marks]

Use the fixed point iteration method to determine a solution accurate within $10^{-2}$ for $x^5 - 3x^3 - 3 = 0$ on the interval [1,2].

[2.5 Marks]

Q2. a) The Lagrangian interpolating polynomial of degree $n$ that passes through $n+1$ data points $(x_0,y_0)$, $(x_1,y_1)$,…………, $(x_{n-1},y_{n-1})$, $(x_n,y_n)$ is defined as $P_n(x) = \sum_{i=0}^{n} y_i L_i(x)$.

where, $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$.

Show that $L_i(x_j) = 1$ when $j = i$ and

$L_i(x_j) = 0$ when $j \neq i$.

[2.0 Marks]

b) Thermistors are used to measure the body temperature and it is based on materials' change in resistance with temperature. To measure temperature, manufacturers provide a temperature vs. resistance calibration curve. A manufacturer of thermistors makes several observations with a thermistor, which are given in following table.

| | | | | |
|---|---|---|---|---|
| $R$ (ohm) | 1102.0 | 921.3 | 634.0 | 453.1 |
| $T$ (°C) | 22.11 | 28.13 | 38.12 | 48.12 |

i) Determine the temperature corresponding to 750.8 ohms using a third order Lagrange polynomial.

[4.0 Marks]

ii) Find the absolute relative approximate error for the third order polynomial approximation. Assume that the second order polynomial approximation is 34.35.

[2.0 Marks]

c) A liquid-liquid extraction process conducted in the Electrochemical Materials Laboratory involved the extraction of nickel (Ni) from the aqueous phase into an organic phase. A typical set of experimental data from the laboratory is given below.

| | | | |
|---|---|---|---|
| Ni aqueous phase, $a$(g/l) | 2.5 | 3.0 | 3.5 |
| Ni organic phase, $g$(g/l) | 7.57 | 9 | 11 |

Assuming $g$ is the amount of nickel in the organic phase and $a$ is the amount of Ni in the aqueous phase, the quadratic interpolant that estimates $g$ is given by

$$g = x_1 a^2 + x_2 a + x_3, 2.5 \le a \le 3.5$$

The solution for the unknowns $x_1$, $x_2$, and $x_3$ is given by

$$\begin{bmatrix} 4 & 2 & 1 \\ 6.25 & 2.5 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7.57 \\ 9 \\ 11 \end{bmatrix}$$

i) Find the values of $x_1$, $x_2$, and $x_3$ using the Gauss-Seidel method.
Use

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

as the initial guess and conduct two iterations.

[4.0 Marks]

ii) Estimate the amount of nickel in the organic phase when 3.3 g/l is in the aqueous phase using quadratic interpolation.

[2.0 Marks]

Q3. a) Write down the first derivative approximation equation for the

i) Forward difference
ii) Backward difference
iii) Central difference

Let $f(x) = e^x$ and using forward difference, backward difference and central differences, approximate $f'(4)$. Use a step size as 0.05.

[3.0 Marks]

b) A trunnion of diameter has to be cooled from a room temperature of 80°F before it is shrink fit into a steel hub. The equation that gives the diametric contraction $(\Delta D)$, in centimeters of the trunnion in dry-ice/alcohol (boiling temperature is $-108$°F) is given by:

$$\Delta D = 12.36 \int_{80}^{-108} \left(-1.23\times10^{-11}T^2 + 6.19\times10^{-9}T + 6.02\times10^{-6}\right)dT$$

i) Use the single segment Trapezoidal rule to find the contraction.
ii) Find the true error, $E_t$, for part (i).
iii) Find the absolute relative true error for part (i).

[5.0 Marks]

c) The concentration of benzene at a critical location is given by

$$c = 1.75\left[erfc(0.65) + e^{31.72} erfc(5.75)\right]$$

where

$$erfc(x) = \int_{\infty}^{x} e^{-t^2}\,dt$$

So in the above formula

$$erfc(0.65) = \int_{\infty}^{0.65} e^{-t^2}\,dt$$

Since $e^{-t^2}$ decays rapidly as $t \to \infty$, we will approximate

$$erfc(0.65) = \int_{5}^{0.65} e^{-t^2}\,dt$$

i) Use two-point Gauss Quadrature Rule to approximate the value of $erfc(0.65)$.
ii) Find the absolute relative true error for part (i) (Assume exact value $= -0.313$).

Refer the following table for the weighting factors and function argument values.

| Point | Weight Factors | Function Arguments |
|---|---|---|
| 2 | $C_1 = 1.0000$ | $t_1 = -0.5773$ |
| | $C_2 = 1.0000$ | $t_2 = 0.5773$ |
| 3 | $C_1 = 0.5555$ | $t_1 = -0.7746$ |
| | $C_2 = 0.8888$ | $t_2 = 0.0000$ |
| | $C_3 = 0.5555$ | $t_3 = 0.7746$ |

[6.0 Marks]

Q4. a) Briefly explain the following and give an example for each item.

i) Ordinary differential equations (ODE)
ii) Partial differential equations (PDE)
iii) Initial value problem (IVP)
iv) Boundary value problem (BVP)

[2.0 Marks]

b) The open loop response, that is, the speed of the motor to a voltage input of 25 V, assuming a system without damping is

$$25 = (0.02)\frac{dw}{dt} + (0.06)w.$$

If the initial speed is zero $(w(0) = 0)$, and using Euler's method, what is the speed at $t = 0.8\text{s}$? Assume a step size of $h = 0.4$ s.

[4.0 Marks]

c) The concentration of salt $x$ in a home made soap maker is given as a function of time by

$$\frac{dx}{dt} = 35.5 - 2.5x$$

At the initial time, $t = 0$, the salt concentration in the tank is 50 $g/l$. Using second order Runge-Kutta method and a step size of $h$=1.5 min determine the salt concentration after 3 minutes?

[4.0 Marks]

d) Use Runge-Kutta's 4th order method to solve, $\frac{dx}{dt} = xy$ for $x = 1.2$ and 1.4. Consider the initial conditions as $x = 1, y = 2$ and use step size $h = 0.2$.

[4.0 Marks]

Q5. a) Classify the following equations as linear or non-linear, and state their order.

i) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$

ii) $\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$

iii) $2\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x \partial t} + 3\frac{\partial^2 f}{\partial t^2} + 4\frac{\partial f}{\partial x} + \cos(2t) = 0$

iv) $\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$

[3.0 Marks]

b) i) List advantages and disadvantages of using the explicit method in solving partial differential equations.

ii) Solve $u_t = 5u_{xx}$ for $0 < x \leq 5$ and $0 < t \leq 0.3$ with $u(0,t) = 0$; $u(5,t) = 60$ and

$$u(x,0) = \begin{cases} 20x & for \quad 0 < x \leq 3 \\ 60 & for \quad 3 < x \leq 5 \end{cases}.$$

Use, $h = 1.0$ and $k = 0.1$, where $h$ and $k$ are step sizes along $x$ and $t$ axes respectively.

[9.0 Marks]

c) Briefly explain the procedure of finite difference solution technique for solving Laplace equation $u_{xx} + u_{yy} = 0$, highlighting any differences with the solution technique used in part (b), section (ii) in above.

[2.0 Marks]