UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2017

Module Number: IS6301 **Module Name: Mathematical Modelling**

[Three hours]

[Answer **all questions**, each question carries fourteen marks]

Q1.

a) i.) Briefly explain the stages of 'Mathematical Modelling Process'.

ii.) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to each solution? What is the added value in each case?

[3 Marks]

b) A brine solution of salt flows at a constant rate of 4 L/min into a large tank that initially held 100 L of pure water. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 3 L/min. Assume that the concentration of salt in the brine entering the tank is 0.2 kg/L. Determine the mathematical expression for the above problem and calculate the concentration of salt in the tank after 30 minutes.

[5 Marks]

c) A train operator for a particular line is buying some new rolling stock. The trains supplied by one company accelerate in such a way that their speed (V_1), in m/s, varies with time (t) in seconds from a standing start according to the expression:

$$V_1(t) = 35\left[\frac{t^2}{1+t^2}\right].$$

However, the expression for the speed (V_2) of trains from a competitor supplier is:

$$V_2(t) = 100\,[1 - e^{-0.02t}].$$

i.) How far will each train travels in 30 seconds from a standing start?

ii.) Which train would you recommend to the operator and why?

[6 Marks]

Q2.

a) i.) What is meant by the statement 'a physical equation is dimensionally homogeneous'?

ii.) Clearly state the 'Buckingham Pi Theorem'.

[2 Marks]

b) A prototype gate valve which will control the flow in a pipe system conveying paraffin is to be studied in a model. The pressure drop across the valve (Δp) is expected to depend upon the gate opening (h), the overall depth (d), the velocity (V), density (ρ) and viscosity (μ). Perform dimensional analysis to obtain the relevant non-dimensional groups.

A 1/5 scale model is built to determine the pressure drop across the valve with water as the working fluid.

i.) For a particular opening, when the velocity of paraffin in the prototype is 5.0 m s^{-1}, what should be the velocity of water in the model for dynamic similarity?

ii.) What is the ratio of the quantities of flow in prototype and model?

iii.) Find the pressure drop in the prototype if it is 50 kPa in the model.

(The density and viscosity of paraffin are 800 kg m^{-3} and 0.002 kg m^{-1} s^{-1} respectively. Assume the kinematic viscosity of water as 1.0×10^{-6} m^2 s^{-1}).

[7 Marks]

c) A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day. Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now suppose that both M and N contain an undesirable drug C, 3 units per gram in M and 1 unit per gram in N. Using graphical method, find the amount of substances M and N (in grams) should be mixed to meet the minimum daily requirements and at the same time minimize the intake of drug C? How many units of the undesirable drug C will be in this mixture?

[5 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of 'Vogel's Approximation Model (*VAM*)' and 'Modified Distribution Method (*MODI*)' used to solve transportation problems.

[2 Marks]

c) A large manufacturing company is closing three of its existing plants and intends to transfer some of its more skilled employees to three plants that will remain open. The number of employees available for transfer from each closing plant is as follows.

Closing Plant	Transferable Employees
1	60
2	110
3	65
Total	235

The following number of employees can be accommodated at the three plants remaining open.

Open Plants	Employees Demanded
A	50
B	75
C	100
Total	225

10 employees available for transfer at closing plant 2 confirmed their early retirement and hence not going to employ at any of the open plants. Each transferred employee will increase product output per day at each plant as shown in the following table. The company wants to transfer employees so as to ensure the maximum increase in product output.

From	To		
	A	B	C
1	5	6	7
2	10	9	12
3	7	10	10

i.) Find the initial solution using *VAM*.
ii.) Solve the problem using *MODI*.
iii.) Are there multiple optimal solutions? Explain. If yes, identify them.

[10 Marks]

Q4.

a) Briefly explain what is meant by,

i.) 'Row reductions' and 'Column reductions' performed in assignment problems.

ii.) 'Prohibited Assignment'.

[2 Marks]

b) Give examples (2 each) for maximization and minimization assignment problems occurring in real world with related to engineering/management.

[2 Marks]

c) A trucking company has one surplus truck in each of the cities; A, B, C, D & E and one deficit truck in each of the cities; 1, 2, 3, 4, 5 & 6. The distance between the cities in kilometers is shown in the table below.

Cities	1	2	3	4	5	6
A	12	10	15	22	18	8
B	18	10	25	15	16	12
C	11	10	3	8	5	9
D	14	6	10	13	13	12
E	8	12	11	7	3	10

If the transportation is prohibited from City C to City 4 due to ongoing road construction works, and City 5 already received a truck from a close-by different city and hence, no longer required a truck from Cities A, B, C, D or E, find the assignment of trucks from the cities in surplus to cities in deficit so that the total distance covered by vehicles is minimum.

Then, find the total minimum distance covered by each vehicle. Are there multiple optimal solutions? If yes, identify them.

[10 Marks]

Q5.

a) Find the equilibrium point, if it exists, for the following dynamical system. Is the equilibrium point stable or unstable?

$$u(n) = 2u(n-1) + v(n-1) + 3$$

$$v(n) = 4u(n-1) - v(n-1) + 6$$

[2.0 Marks]

b) Consider the following first-order recurrence relation,

$$X_{n+1} = F(X_n) = X_n^2 - a$$

where a is a constant.

i.) If $a = 0$, find all fixed points and determine their stability.

ii.) For what range of values of a does the recurrence relation have two fixed points?

iii.) If the recurrence relation has two fixed points, for what range of values of a are both unstable?

iv.) If the recurrence relation has a cycle of length 2, such that $X_0 = X_2 = X_4 = \ldots\ldots$ and $X_1 = X_3 = X_5 = \ldots\ldots$, then show that $X_n + X_{n+1} = -1$ for all $n \in \mathbb{N}$.

[6.0 Marks]

c) The number of rabbits on an island is currently 600, and it is increasing by 7.5% per year. Let R(t) represent the number of rabbits on this island t years from now.

i.) Write down a discrete dynamical system along with an initial value for R(t).

ii.) Find an explicit formula for R(t).

iii.) How many years will it take for the rabbit population on this island to double in size?

[6.0 Marks]