UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: July 2017

Module Number: CE7203 **Module Name: Computer Analysis of Structures**

[Three Hours]

[Answer **all questions**, marks are given as indicated]

Use Separate booklets for Section A and Section B

SECTION A

Q1. Fig. Q1 shows a one dimensional spring assembly. Element numbers are boxed and node numbers are circled. Element 1 has stiffness of 3K, while elements 2 and 5 has 2K each. Rest of the elements (i.e. 3 and 4) has stiffness of 1K each.

a) Assemble the stiffness matrix of the system of springs following the node numbering given in Fig. Q1.
Note: Use the same node numbering and degrees of freedom resulted from the node numbering above in the assembly of the global stiffness matrix of the system.

[4 Marks]

b) Calculate the deformations in nodes 2 3 and 4 and hence the resisting force at fixed end (i.e. Node 1)

[3 Marks]

c) Determine the equivalent spring for all the spring elements and compute the displacement of the equivalent spring under the applied resultant force. Compare the overall displacement of the system of spring (i.e. displacement of Node 4) with the displacement of the equivalent system and explain the reason why the two displacements are different to each other.

[2 Marks]

d) Comment on the selection of node numbering adopted the influence of node numbering on the stiffness matrix and the final outcome of the Finite element analysis.
(You may use different node numbering to support your explanations)

[1 Mark]

Q2. Water fountain is to be set up at the center of a kiddie's pool 4 m above the pool level. Proposed supporting structure consists of 4 members in a form of a space truss shown in Fig. Q2. Coordinates of the nodes according to a rectangular Cartesian coordinate system with node A as the origin (0,0,0) is given in Fig. Q2. All joints are to be considered as pin-joints.

a) i) Identify the degrees of freedom associated with each member and formulate individual element matrices for the four members (Assuming that all four members have identical stiffness characteristics, i.e. Identical Young's modulus (E) and cross-sectional area (A))

[4 Marks]

ii) Assemble the global stiffness matrix

[2 Marks]

b) In terms of the weight of the fountain, P, and stiffness of the members, EA, find the displacement at point E.

[2 Marks]

c) Determine the member forces and explain the reasons for recording identical member forces in all the members.

[2 Marks]

Q3. a) Assuming a third order polynomial displacement function, explain the deformation shape of an element. Show that the stiffness matrix for a beam element with stiffness EI and length L is as indicated below. Ignore the axial effect and consider only rotational and translational degrees of freedom of the beam in the derivation of the stiffness matrix.

$$[k^e] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

[6 Marks]

b) Fig Q3 shows a continuous beam ABC, fixed at A and C and held on a roller support at B. Beam is loaded midway between span AB and BC by a point load P kN as shown in Fig. Q3. Using suitable finite element formulation to accommodate loading on the beam, calculate deformation of the beam under the applied load P. Assume bending stiffness of the beam, EI, and remain constant though out the length.

[4 Marks]

SECTION B

Q4. a) Identify the governing criteria of matrix flexibility method and explain how it affects on the analysis process.

[2 Marks]

b) i) Identify the type of structure shown in Fig. Q4, and hence determine its static indeterminacy (SI°).

[3 Marks]

ii) Using matrix flexibility method, determine all the member forces and the nodal displacement at node 'B' for the structure shown in Fig. Q4. Assume *AE* is constant for all the members, where *A* is the cross-sectional area and *E* is the elastic modulus.

[10 Marks]

Q5. a) Briefly explain the degree of kinematic indeterminacy (KI°) of a structure.

[2.5 Marks]

b) An idealized two-dimensional frame structure is shown in Fig. Q5. Both members have constant flexural rigidity of EI and the axial deformations are considered negligible. Vertical and horizontal point loads with magnitude P have been applied at the node 2 as shown in Fig. Q5. General element stiffness matrix for a beam element with usual notations is as follows.

$$
[k] = \begin{bmatrix}
\frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
$$

(i) Determine the degree of kinematic indeterminacy (KI°) of the structure shown in Fig Q5.

[2.5 Marks]

(ii) Using matrix stiffness method, determine the deformations (i.e. displacement and rotation) at the node 2.

[10 Marks]

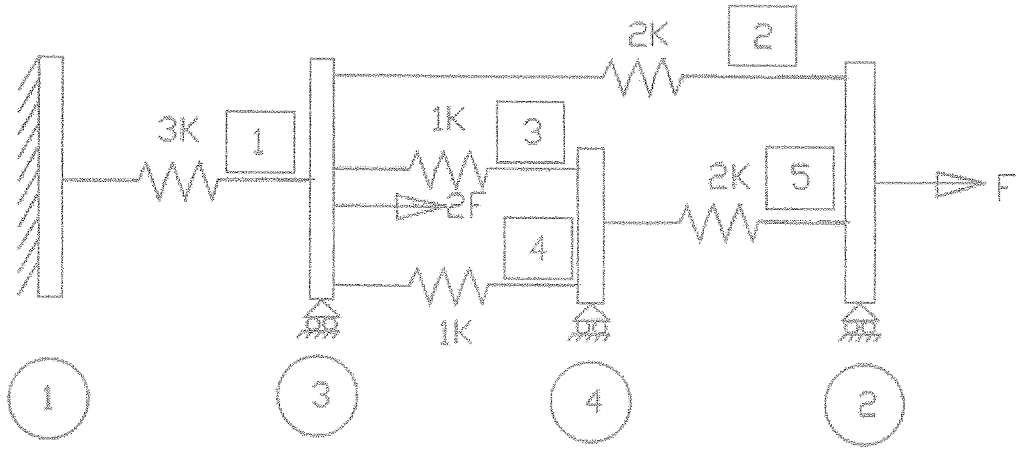

Fig. Q1

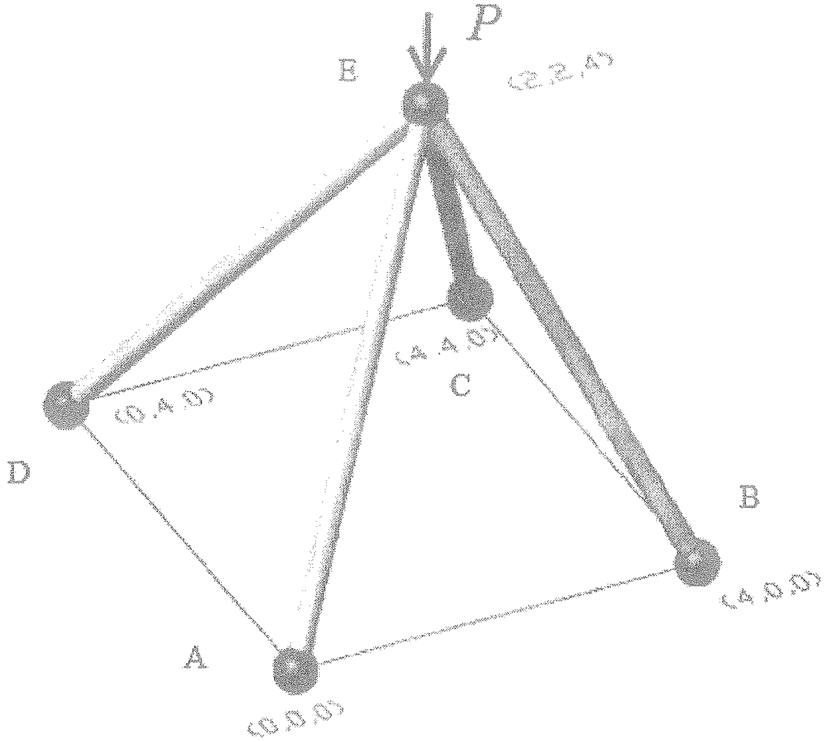

Fig. Q2

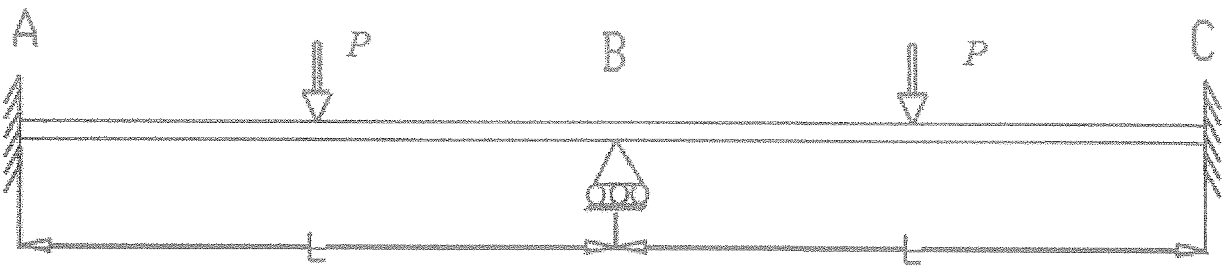

Fig. Q3

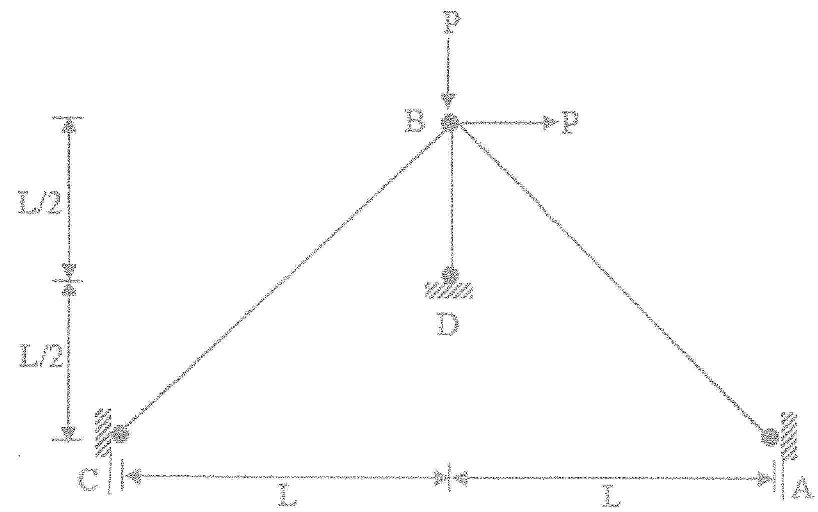

Fig. Q4

Note: Joints A, B, C, and D, are pinned.

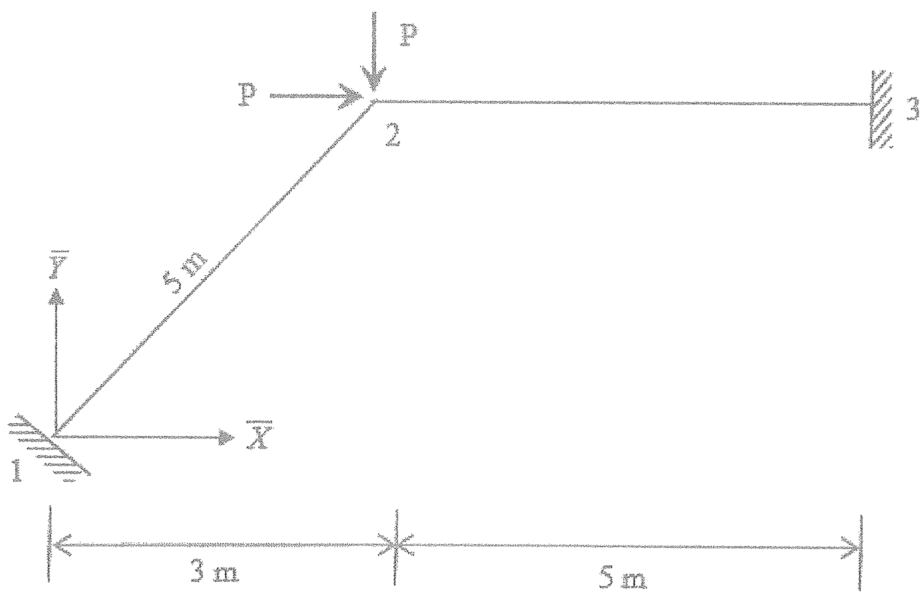

Fig. Q5