UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: July 2017

Module Number: EE7209 **Module Name: Digital Signal Processing**

[Three Hours]

[Answer all questions, each question carries 10 marks]

Q1 a) A Linear-Time Invariant (LTI) system is described by

$$y[n] = \sum_{k=-\infty}^{n+1} x[k]$$

Check the system regarding the following properties.

i) Dynamicity
ii) Linearity
iii) Time invariance
iv) Causality
v) Stability

[4 Marks]

b) An LTI system has the impulse response $h[n] = u[n]$. Determine the output of this system for the input $x[n]$ shown in Figure Q1 and described as

$$x[n] = \begin{cases} 0 & n < 0 \\ a^n & 0 \le n \le N_1 \\ 0 & N_1 < n < N_2 \\ a^{n-N_2} & N_2 \le n \le N_2 + N_1 \\ 0 & N_2 + N_1 < n \end{cases}$$

where $0 < a < 1$.

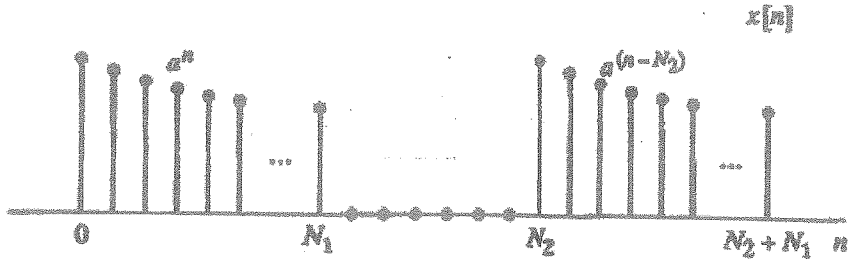

Figure Q1

[6 Marks]

Q2 a) Briefly explain one of the inverse Z-transform techniques.

[3 Marks]

b) When the input to a Linear-Time Invariant (LTI) system is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1]$$

the output is

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$$

i) Determine the system function $H(Z)$ of the system by indicating the region of convergence.

ii) Determine the impulse response $h[n]$ of the system.

iii) Determine the difference equation of the system that relates the output $y[n]$ to the input $x[n]$.

iv) Is this system stable? Justify your answer.

v) Is this system causal? Justify your answer.

[7 Marks]

Q3 a) Explain explicitly the difference between the Fourier transform of a discrete-time sequence (DTFT) and the Discrete Fourier Transform (DFT) of the same discrete-time sequence.

[3 Marks]

b) Consider the finite length sequence $x[n]$ shown in Figure Q3.

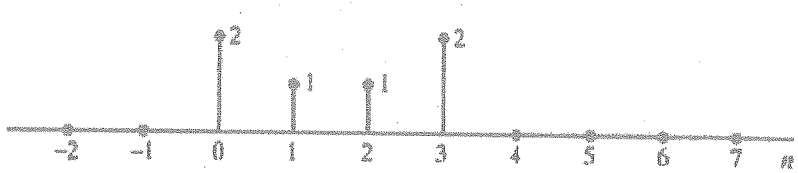

Figure Q3

i) Compute the 5-point DFT of $x[n]$ denoted by $X(k)$.

ii) Determine the sequence $y[n]$ whose DFT is

$$Y(k) = W_5^{-2k} X(k)$$

where $W_N = e^{\left(-j\frac{2\pi}{N}\right)}$

[7 Marks]

Q4 a) Figure Q4 shows a butterfly stage extracted from a signal flow graph for implementing a Fast Fourier Transform (FFT) algorithm. Choose and justify the most accurate statement from the following.

A) The butterfly was extracted from a decimation-in-time FFT algorithm.

B) The butterfly was extracted from a decimation-in-frequency FFT algorithm.

C) It is not possible to say from the figure which kind of FFT algorithm that the butterfly stage is used.

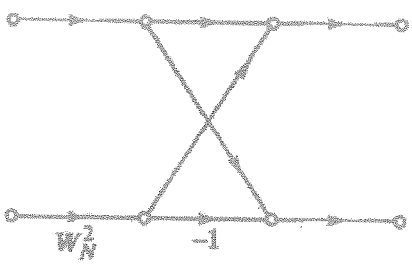

Figure Q4

[3 Marks]

b) Compute the 8-point DFT of the sequence

$$x[n] = \sin\left(\frac{n\pi}{2}\right) \qquad 0 \le n \le 7$$

using the radix-2 decimation-in-time FFT algorithm.

Note:
Show all the computations clearly at each intermediary stage.

[7 Marks]

Q5 a) Briefly explain the difference between the Impulse Invariance method and the Bilinear Transform method used for the digital filter design in Infinite Impulse Response (IIR) systems.

[3 Marks]

b) Consider a causal Linear-Time Invariant (LTI) system S described by the system function

$$H(Z) = \frac{\left(1-2Z^{-1}\right)\left(1-4Z^{-1}\right)}{\left(1-\frac{1}{2}Z^{-1}\right)\left(1+\frac{1}{4}Z^{-1}\right)}$$

i) Draw a direct form II signal flow graph for the Systems S.

ii) Draw the parallel form structure for the system S by determining the partial fractions with first-order systems.

[7 Marks]