UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: March 2021

Module Number: CE7203 **Module Name: Computer Analysis of Structures**

[Three Hour]

[Answer all questions. Each question carries marks as indicated]

Q1. A steel truss structure used to support a crane system in a factory building is shown in Fig. Q1. For a safety concern, the structural engineer wants to determine the reaction forces at the supports and the member forces in all the elements while the maximum load of 24 kN is raised by the crane fixed to the node 'D'. The cross-sectional area, A, for all the elements is 1000 mm^2 and the elastic modulus, E, for steel is 200 GPa.

a) In order to use the force method of analysis, determine degree of statical indeterminacy, SI° for the structure.

[2 Marks]

b) Using matrix flexibility method, determine member forces in all the element in the structure. [**Hint**: Depending on the degree of SI°, consider redundant as member/s force in element with descending order].

[10 Marks]

c) Determine the vertical deflection at node 'D' in millimetres.

[3 Marks]

With usual notations, member flexibility matrix for a truss element is [L/AE].

Q2. An idealized concrete frame structure is shown in Fig. Q2. The frame consists of two members (i.e., a beam and a column) of constant flexural rigidity, EI, connected by a rigid joint at node B. The member BC supports a concentrated load P acting downward at midspan and the member AB carries a uniformly distributed horizontal load of intensity w as shown in Fig Q2. The magnitude of w (in units of load per unit length) is equal to 3P/L. The change in length of the members can be neglected.

a) Explain briefly how the degree of kinematic indeterminacy (KI°) affects the computing time in matrix stiffness method of analysis. What is the KI° for the given structure

[2 Marks]

b) In order to analyse this structure using matrix stiffness method, convert the given structure into nodal load structure.

[3 Marks]

c) Determine support reactions and member end forces in the beam and the column.

[10 Marks]

With negligible axial effect, the element stiffness matrix for a beam element is given at the end of Q5.

Q3. Fig. Q3 shows a one dimensional spring assembly. Element numbers are boxed and node numbers are circled. k_1 = 100 kN/m, k_2 = 200 kN/m and k_3 = 300 kN/m, where k_1, k_2 and k_3 are stiffness of the spring elements 1, 2 and 3 respectively. Face A-A is fixed and it is 5 mm from Node 3. Consider load applied at Node 2 as **P** = 1500 N.

a) Assemble the global stiffness matrix of the system of springs.

[3 Marks]

b) Determine the displacements at Nodes 2 and 3. Assess whether the bar at Node 3 has any contact with the face A-A.

[2 Marks]

c) Determine the minimum required force **P** at Node 2 to make Node 3 contact with the face A-A?

[2 Marks]

d) If P = 2000 N, determine the support reaction at face A-A?

[2 Marks]

e) If the spring element [1] in Fig Q3 replace with bar element (Young's modulus **E**, cross-section area **A** and length **L**), explain the procedure of analysising the above problem using FEM?

[1 Mark]

Q4. Pin-jointed 2D truss is pinned supported at Node **A** and roller supported at Nodes **B** and **C** as shown in Fig Q4. The Young's modulus E = 200 GPa for all three elements and cross-section area A = 6×10^{-4} m^2 for element (1) and (2), $6\sqrt{2} \times 10^{-4}$ m^2 for element (3). The truss system is subjected to a horizontal force of 1000 kN at Node **B**.

a) Write the element stiffness matrix of the 3 elements with respect to a selected global coordinate system.

[3 Marks]

b) Determine the global stiffness matrix of the system.

[1 Mark]

c) Define the boundary condition and loading condition for each node.

[2 Marks]

d) Determine the displacement at Nodes **B** and **C**.

[2 Marks]

e) Determine the support reactions at each node.

[2 Marks]

(Use the stiffness matrix for a 2D-bar element as shown below.)

$$\left[k^e\right] = \frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

where $c = Cos\Theta$, $s = Sin\Theta$ and Θ is the anticlockwise angle at node measured from the global X-axis to the local x-axis of the bar element.

Q5. a) A cantilever beam is loaded with a uniformly distributed load throughout the beam having a length of L. Explain how these loading can be converted for FEM analysis?

[2 Marks]

b) A beam subjected to different loading is shown in Fig. Q5 and is supposed to be analysed using the finite element technique with three elements as shown in Fig Q5. The Young's Modulus of the beam is E, the second moment of area is I. Consider $EI = 20 \times 10^6$ N/m².

i) Determine the equivalent nodal forces and moments at the four nodes of the beam and draw the resultant on the model.

[2 Marks]

ii) Determine the nodal displacements and rotations.

[6 Marks]

(Ignore the axial effect and use the stiffness matrix for a beam element, with usual notations, as shown below.)

$$[k^e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

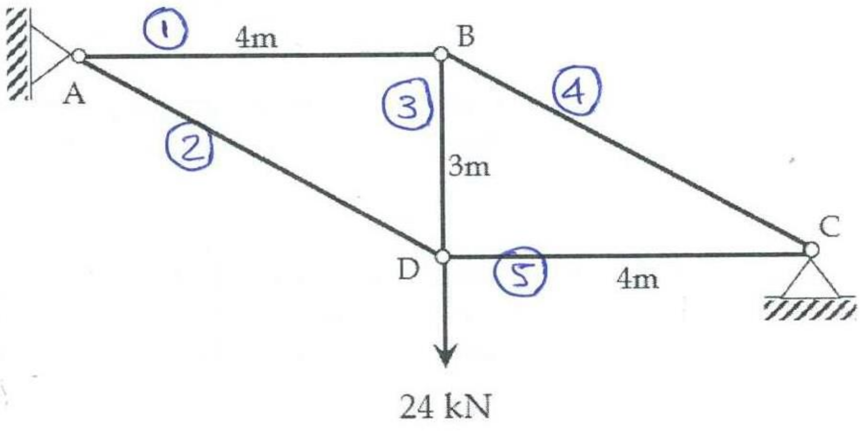

Fig. Q1

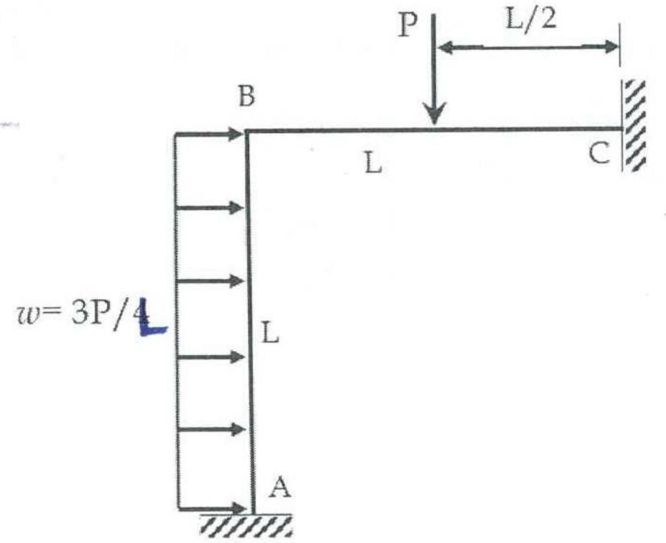

Fig. Q2

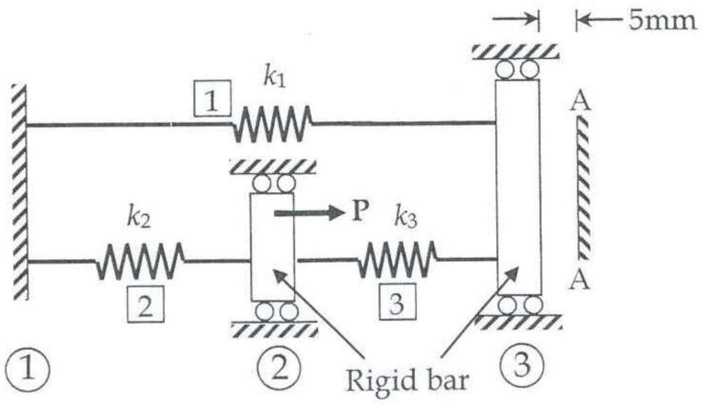

Fig. Q3

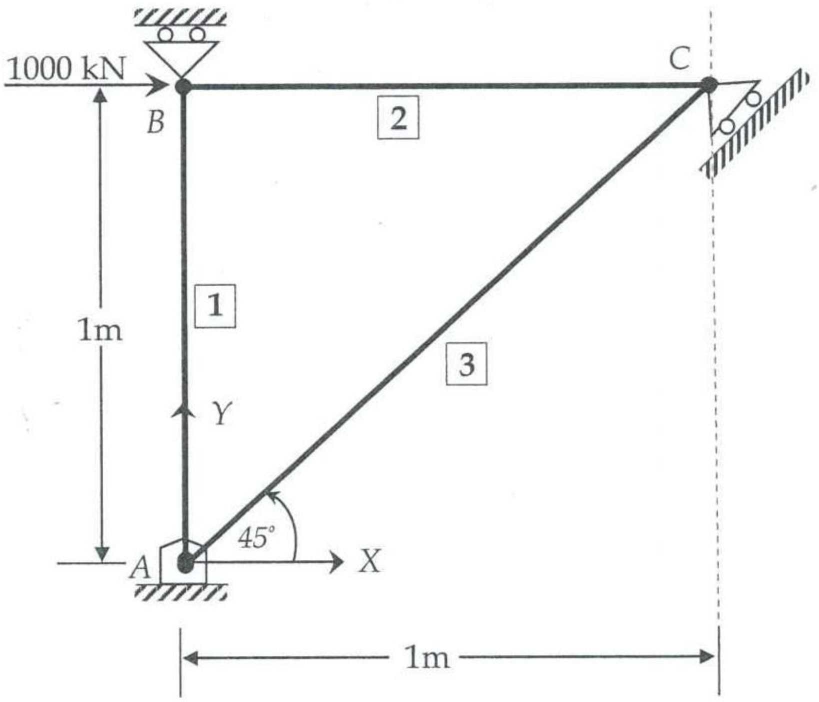

Fig. Q4

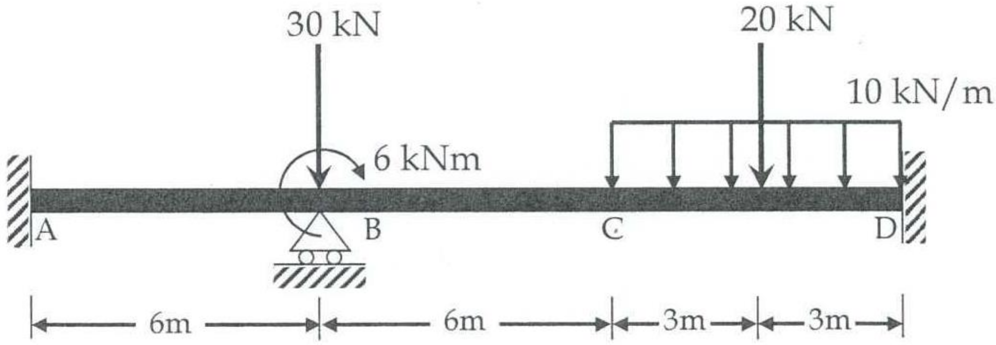

Fig. Q5