# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2021

**Module Number: CE3204** **Module Name: Structural Analysis I**

**[Three Hours]**

[Answer **all question**, each question carries twelve marks]

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Q1 Beam AB, 8 m long, spanning between two fixed supports are shown in Fig Q1. Uniformly distributed load of 25 kN/m is applied on the beam. The beam has a uniform cross section with constant stiffness EI. Calculate the support reaction of the beam independently based on the following methods. (EI =4.2x10$^5$ kNm$^2$)

a) Using simple beam theory

[4 Marks]

b) Using virtual work assuming a suitable virtual force system.

[4 Marks]

c) Using Castigliano's second theorem, theorem of complimentary strain energy with complementary strain energy replaced by strain energy due to linear elastic nature of the problem.

[4 Marks]

Q2 Fig. Q2 show a continuous beam ABCD with symmetrical geometry and arrangement of loading. Beam ABCD has uniform cross section and uniform rigidity EI=4.2x10$^5$ kNm$^2$.

a) Using moment distribution method calculate the joint moments at joints A, B, C and D.

[6 Marks]

b) Using principal of superposition complete the bending moment diagram of the beam ABCD

[2 Marks]

c) Using equilibrium equation, calculate all the support reactions of the beam ABCD and complete the shear force diagram.

[4 Marks]

Q3 a) State two theories involved with the formulation of Three Moment Theorem (TMT) in your own words. Derive the three moment theorem based on the above theories use sketches (if any) required to clarify your explanations.

[3 Marks]

b) A straight elastic beam element ABC with constant flexural rigidity EI =4.2x10$^5$ kNm$^2$ is used as a ground floor tie beam in a multi storied building structure is shown in Fig. Q3. Joints A,B and C is applied with point loads 750kN, 1250kN and 1000 kN respectively transferred from columns from upper floors. Spring constants at support A,B, and C calculated from the soil structure interaction is estimated to be 11,250 kN/m. It is expected that supports will settle based on the magnitude of the final reactions at A, B and C. Assume that the beam is straight before loading and settles based on the magnitude of the final reaction.

(i) Based on the conditions described above and using the Three Moment Theorem, calculate the moment at support with due consideration that beam will settle based on the magnitude of the reaction.

$$\frac{M_1 l_1}{E_1 I_1} + 2M_2\left(\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right) + \frac{M_3 l_2}{E_2 I_2} = 6\left[\left(\frac{\delta_1 - \delta_2}{l_1} + \frac{\delta_3 - \delta_2}{l_2}\right) - \left(\frac{A_1 a_1}{E_1 I_1 l_1} + \frac{A_2 a_2}{E_2 I_2 l_2}\right)\right]$$

[6 Marks]

(ii) Draw the bending moment and shear force diagram of the beam ABC under the maximum applied load.

[3 Marks]

Q4 Fig. Q4(a) shows an idealized portal frame structure ABC constructed as a shade of car park. The column of the portal frame is fixed at the ground while free end of the beam kept sliding on top of an existing structure. The only load on the structure is considered to be the roof load applied on the beam. The rotational stiffness EI of the column and beam are $4x10^4$ kNm$^2$ and $8x10^4$ kNm$^2$ respectively.

a) Considering the uniformly distributed load on the beam from the roof $w$=10 kN/m, calculate the rotations at the joints B and C using slope deflection relationship.

[6 Marks]

b) Based on the rotations calculated in part (a), determine the moments at joints A, B and C and hence complete bending moment and shear force diagrams of the frame ABC.

[3 Marks]

c) In an event that the column is pined to the ground at joint A (Fig. Q4(b)), explain how the calculation of the above moment would be different.

[3 Marks]

Q5 Fig. Q5 shows plan view of a cantilever beam ABC subjected to a 20kN point load and 50kNm torsional moment at the cantilever tip. Ignore the self-weight of the beam. Consider the rotation stiffness EI of the beam to be $4x10^4$ kNm$^2$ and torsional rigidity of the beam GJ $5x10^4$ kNm$^2$.

a) Considering only the torsional and bending energy calculate the downward deformation of the cantilever tip C using the energy theory.

[6 Marks]

c) Considering the fact that the cantilever beam and hence a determinate structure and that the real deformation of a determinate structure can be found by assuming a self-equilibrating virtual force set, calculate the vertical displacement of the cantilever tip C using virtual work and show it is equal to Q5 a) above. Assume work done due to torsional moment similar to work done by moment and is equal to multiple of torsional moment into torsional rotation.

[6 Marks]

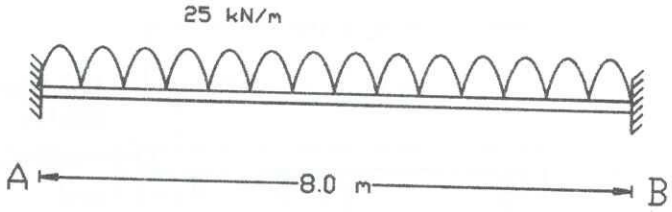



Fig. Q1

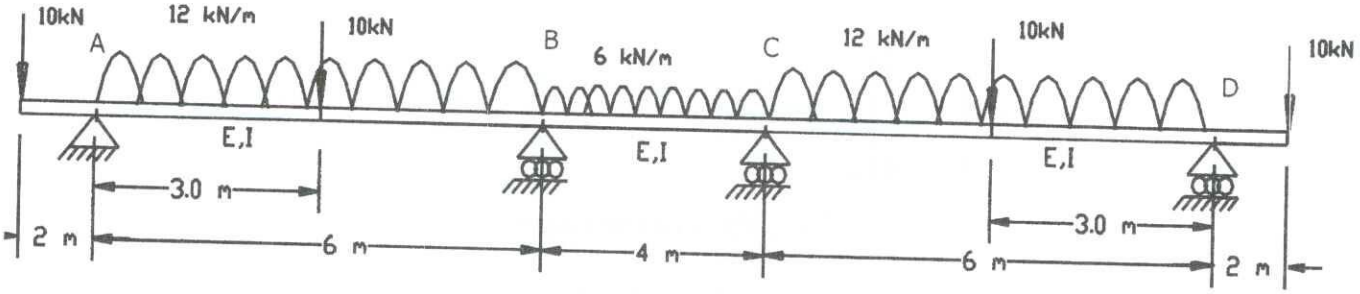



Fig. Q2

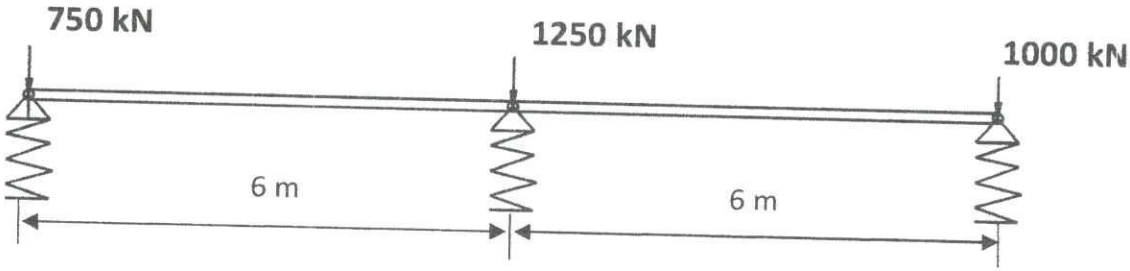



Fig. Q3

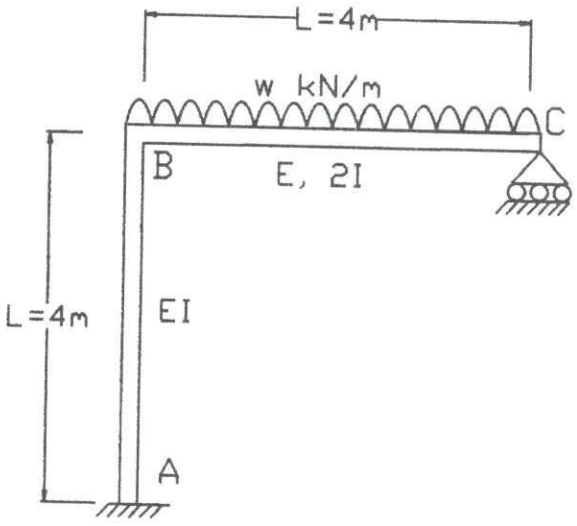



Fig. Q 4 (a)

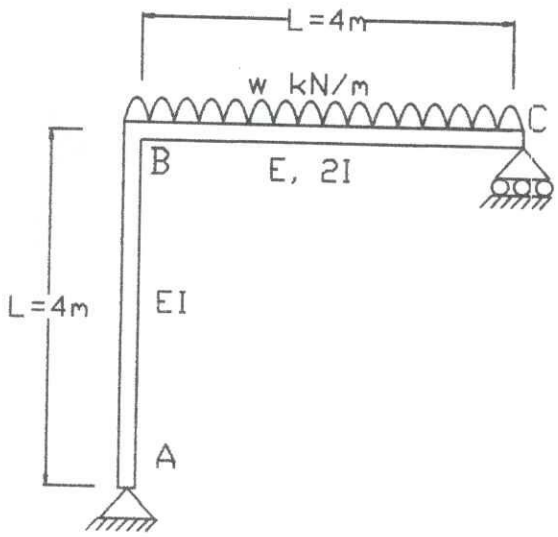
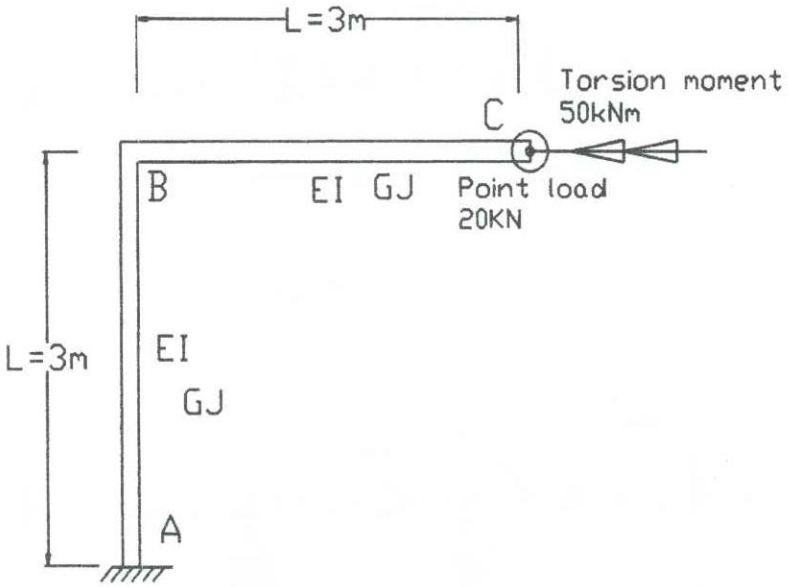



Fig. Q4 (b)

Fig. Q4




Plan View of the beam

Fig. Q5