UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2021

Module Number: EE3205 **Module Name: Signals and Systems**

[Three Hours]

[Answer **all questions**, each question carries 10 marks]

Q1 a) i) State a mathematical expression to determine the energy of a continuous time signal.

[2 Marks]

ii) An arbitrary real valued continuous time signal is represented by

$$f(t) = f_e(t) + f_o(t)$$

where $f_e(t)$ and $f_o(t)$ are the even and odd components of $f(t)$. The signal $f(t)$ occupies the entire interval $-\infty < t < \infty$. Show that the energy of the signal $f(t)$ is equal to the sum of the energy of the even component $f_e(t)$ and the energy of the odd component $f_o(t)$.

[2 Marks]

b) The impulse response of a continuous time LTI system is given by

$$h(t) = u(t+1) - u(t-3).$$

i) Is this system causal?

[1.5 Marks]

ii) Is this system stable?

[1.5 Marks]

iii) Find and sketch the system response to the input

$$f(t) = \delta(t-1) - 2\delta(t+1).$$

[3 Marks]

Q2 a) Fourier Series (FS), Discrete Time Fourier Series (DTFS), Fourier Transform (FT) and Discrete Time Fourier Transform (DTFT) are the four distinct Fourier representations for different class of signals. State the appropriate Fourier representations (i.e. FS, DTFS, FT or DTFT) for the following signals. Justify your answers.

i) $f(t) = 1 - \cos(2\pi t) + \sin(3\pi t)$

[2 Marks]

ii) $f[n] = \left(\frac{1}{2}\right)^n u[n]$

[2 Marks]

b) Consider the LTI system shown in Fig. Q2.

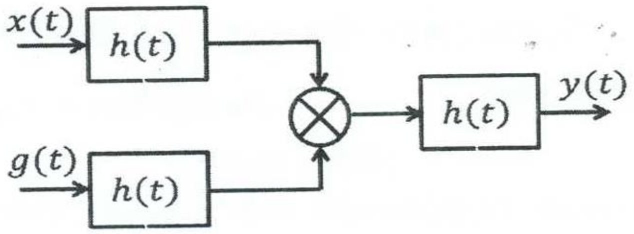

Fig. Q2

The impulse response of each sub system is $h(t) = \frac{\sin(11\pi t)}{\pi t}$ and the two inputs to the system are $x(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos(k5\pi t)$ and $g(t) = \sum_{k=1}^{10} \cos(k8\pi t)$.

i) Determine the Fourier transforms of $h(t)$, $x(t)$ and $g(t)$. Refer the Fourier transform pairs shown in Table 1.

[3 Marks]

ii) Using the results obtained in part b) i) determine the overall system output $y(t)$.

[3 Marks]

Q3 a) The Unilateral Laplace transform of a signal $f(t)$ is given by $F(s)$.

i) Show that the Laplace transform of $f(t)e^{-at}$ is $F(s-a)$.

[1 Mark]

ii) Determine the Laplace transform of $z(t) = e^{-at}u(t)$. Show all the mathematical derivations in your answer.

[2 Marks]

iii) Using the results obtained in part a) ii), determine the Unilateral Laplace transform of $y(t) = e^{-at}\cos(\omega_0 t)u(t)$.

[2 Marks]

b) Consider the circuit shown in Fig. Q3. Assume that the initial conditions are $i_L(0) = 1$ A and $v_c(0) = 3$ V .

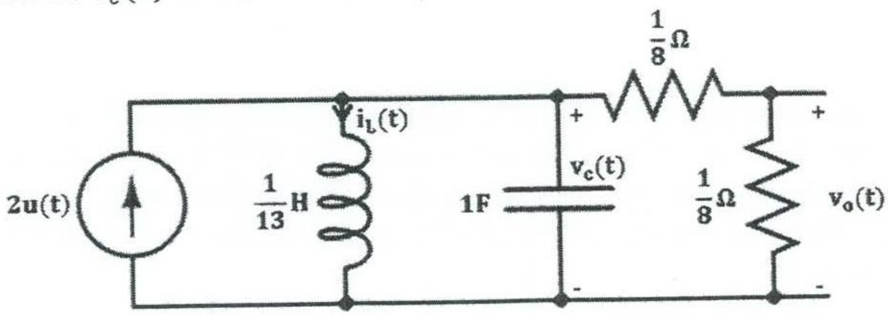

Fig. Q3

i) Draw the transformed circuit in s-domain by representing all the voltages and currents by their Laplace transforms.

[2.5 Marks]

ii) Determine the output voltage $V_o(s)$ in s-domain.

Hint: A capacitor C with an initial condition $v_c(0)$ can be represented by an uncharged capacitor of impedance $\frac{1}{Cs}$ in parallel with a current source $v_c(0)$. Similarly, an inductor L with an initial current $i_L(0)$ can be represented by an inductor of impedance Ls in parallel with a current source $\frac{i_L(0)}{s}$.

[2.5 Marks]

Q4 a) i) Briefly explain why discrete-time signal processing is required for continuous time signals.

[2 Marks]

ii) Explain why low pass filtering is needed before obtaining samples of a continuous time signal.

[2 Marks]

b) Suppose a signal $f(t)$ is uniquely represented by a discrete sequence

$$f[n] = f(nT_s)$$

where T_s is the sampling interval. Determine the conditions to be satisfied on the T_s for the following signals.

i) $$f(t) = \frac{\sin(10\pi t)}{\pi t}$$

[3 Marks]

ii) $f(t) = \cos(\pi t) + 3sin(2\pi t) + \sin(4\pi t)$

[3 Marks]

Q5 a) i) State a mathematical expression for the z-transform of a discrete time signal $f[n]$.

[2 Marks]

ii) Determine the z-transform of $\left(\frac{1}{3}\right)^n u[n]$.

Hint: $\sum_{i=0}^{n} a^n = \frac{1}{1-a} \quad |a| < 1.$

[3 Marks]

b) i) State the mathematical expression for the complex exponential Fourier Series having a fundamental frequency ω_0 and a set of Fourier coefficients $\{a_k\}$.

[2 Marks]

ii) Determine the Fourier Series representation of the signal

$$f(t) = 3\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$$

[3 Marks]

Table 1: Common table of Fourier transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	