UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: December 2020

Module Number: CE 5204 (N/C) **Module Name: Structural Analysis III**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) Explain briefly the different types of yield lines?

[1 Mark]

b) Discuss briefly how you establish a yield line pattern for a given slab.

[3 Marks]

c) A regular polygonal slab of '6' sides, which is isotropically reinforced and simply supported along the edges. The perimeter length of the slab was found as 'L'. The slab carries a uniformly distributed load of intensity 'p' per unit area.

i) Draw a possible yield line pattern at collapse.

[2 Marks]

ii) Determine the corresponding collapse load, assuming the yield moment per unit length of slab is 'm'.

[6 Marks]

Q2. a) Discuss briefly the load resistance mechanism (s) of a rectangular plate

[3 Marks]

b) A thin rectangular plate of side dimensions '$2a$', 'a' and thickness't' is shown in Figure Q2. The plate is simply supported along all four edges. It is subjected to a vertical downward load of intensity,

$$p(x,y) = p_o sin\frac{\pi x}{2a} sin\frac{\pi y}{a}$$

Where, p_o is a constant.

i) Assuming a trail solution for displacement, show that the trail solution satisfies the relevant displacement and boundary conditions.

[3 Marks]

ii) Determine deflection of the plate. Hence, determine bending moment and shear forces on the plates.

[6 Marks]

Governing equation and the equations for bending moments and shear forces (with usual notations and sign convention) are given by

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

$$Q_x = -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) \qquad Q_y = -D\left(\frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^3 w}{\partial y^3}\right)$$

where ,

$$D = \frac{Et^3}{12(1-\nu^2)}$$

Q3. a) What are the assumptions in deriving the governing equation for a thin circular plate?

[3 Marks]

b) A circular plate of radius, r_o, with a concentric hole of radius, r_h, is fixed along the inner boundary and free along the outer boundary. The plate, manufactured using a material having a poison ratio of 0.3, is used to be resisted a uniformly distributed vertically downward load of 'q' per unit area. For a value of r_o/r_h=4

i) Determine the bending moment at the fixed edge, by considering a radial strip as a beam with the loading and end connection as in the plate.

[2 Marks]

ii) Determine the exact radial moment at the fixed edge.

[4 Marks]

iii) Evaluate the suitability of the solutions determined in Part b(i) and Part b(ii) for the designing of the plate.

[3 Marks]

Governing equation and the equation for the radial moment of circular plate (with usual notations and sign convention) are given by

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\frac{dw}{dr})\right] = \frac{Q}{D} \quad M_r = -D\left(\frac{d^2 w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) \quad M_t = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2 w}{dr^2}\right)$$

where

$$D = \frac{Et^3}{12(1-\nu^2)}$$

Q4. a) Show that the membrane stresses in a cylindrical shell (*with usual notations and sign convention*) are given by

$$\frac{N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\phi x}}{\partial \phi} + X = 0 \qquad \frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + Y = 0 \qquad \frac{N_\phi}{R} + Z = 0$$

[6 Marks]

b) Figure 4 shows a semi-circular cylindrical shell made of thin steel sheets, which is proposed to be used as a cantilever roof of a bus stop. The length and the radius of the roof are L and r, respectively. Assume that the load acting on the shell is its self-weight of 'p' per unit surface area.

Based on the membrane theory obtained in Q4 Part (a), determine membrane stress resultant in the roof shell structure. Clearly state if any assumptions are made.

[6 Marks]

Q5 a) Explain briefly the classification of shells.

[2 Marks]

b) Show that the membrane stresses in a conical shell (*with usual notations and sign convention*) are given by

$$N_\theta = P_r\, S \tan\alpha$$

$$N_S = \frac{1}{S}\int (P_r\, S \tan\alpha - P_s\, S)\,ds$$

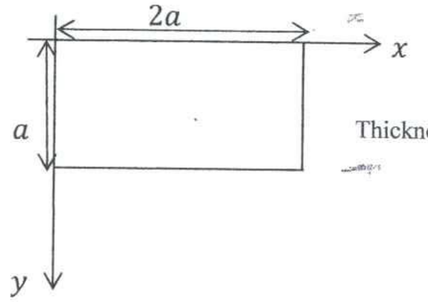
Assume that the membrane stresses in a spherical shell (with usual notations and sign convention) are given by

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = P_r \qquad P_\phi r r_1 - r_1 N_\theta Cos\phi + \frac{\partial(rN\phi)}{\partial\phi} = 0$$

[4 Marks]

c) A conical shell made of thin aluminum sheets is proposed to be used as a roof of a cafeteria as shown in Figure Q5. The shell is subjected to its self-weight of *"p"* per unit surface area.
From the membrane theory derived in Part (b), determine the membrane stress resultant in the roof shell structure.

[6 Marks]



Thickness of the plate is t

Figure Q2

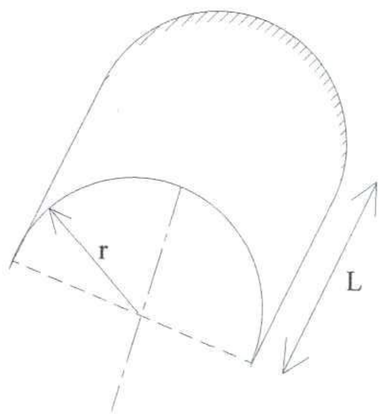


Figure Q4

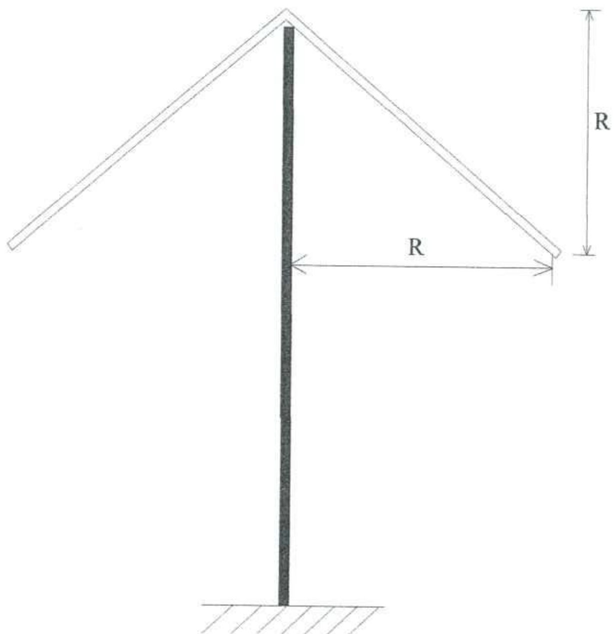


Figure Q5