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An approach towards settling a conjecture on ample numbers

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A positive integer n is said to be *abundant* if $\sigma(n) > n$ and *ample* if $a(n) > n$, where $\sigma(n)$ denotes the sum of positive divisors of n and $a(n)$ denotes the number of recursive divisors of n. In 2005, Iannuci claimed that there exist abundant numbers that are not divisible by the first k primes for all $1 \leq k \leq 7$. The conspicuous parallel properties between abundant and ample numbers have led to the conjecture, *there exist ample numbers that are not divisible by the first k primes for all k*. In 2020, Fink used computational evidence to find the smallest ample number that is not divisible by the first two primes as 3.3×10^{81}. In the present study, we consider a positive integer x that is not divisible by the first k primes for all $k \in \mathbb{N}$ and write $x = q_1^{\alpha_1} q_2^{\alpha_2} \ldots q_j^{\alpha_j};\ q_j = p_{k+j}$, $j \geq 1$ and $0 \leq \alpha_j < \infty$. We try to construct a general formula for $a(x)$ by considering the types of proper divisors m_i of x and formulating the total number of proper divisors $a(M_i) = \sum a(m_i)$, where M_i is the set containing all the proper divisors of type m_i. We show that $a(M_1) = 2\sum_{i=1}^{j}[2^{\alpha_i} - 1]$, where $M_1 = \{q_i^{\alpha_k} : i = 1, 2, \ldots, j\ ;\ \alpha_k = 1, 2, \ldots, \alpha_i\}$ and $a(M_2) = 3j[j-1]$, where $M_2 = \{q_i q_l : i = 1, 2, \ldots, j-1\ ;\ l = 2, 3, \ \ldots, j\ \}$ with j being the number of distinct primes in the prime factorization of x. The aforementioned results are used in the ongoing study on settling the conjecture which states such a positive integer x is ample. These ideas can be utilized in finding new possible modular grid sizes and related applications therein.

Keywords: Abundant numbers, Ample numbers, Recursive divisibility

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