UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: November 2017

Module Number: ME 8214 **Module Name: Robot Manipulator Dynamics**

[Three Hours]

[Answer all questions, each question carries **fifteen** marks]

You may make additional assumptions where necessary, but clearly state them in your answers. Symbols stated herein denote standard parameters.

Q1. a) Derive basic rotation matrices for $R_x(\theta)$, $R_y(\beta)$, $R_z(\gamma)$ using scalar product method where $R_x(\theta) - \theta$ rotation about x axis.

[3.0 Marks]

b) {A} and {B} are two coincident frames. Frame {B} rotates 30^0 about Z_A , 45^0 about X_A, and then translates to (3, 2, 1) w.r.t. frame {A}.

I. Determine ${}^A_BR$, ${}^AP_{Borg}$, and ${}^A_BT$.

[5.0 Marks]

II. A vector ${}^BP = \{1, 1.5, -3\}$ is attached to {B}. Determine ${}^AP$ the position coordinates of P w.r.t. {A}.

[2.0 Marks]

III. Determine ${}^B_AT$ without using inverse matrix transformation.

[3.0 Marks]

IV. A vector ${}^AQ = [1.5, 0, -2]$ is attached to frame {A}. Determine ${}^BQ$.

[2.0 Marks]

Q2. A robot manufacturing work cell is shown in Figure Q2.

a) Determine ${}^1_2R$, ${}^2_3R$, ${}^3_4R$, ${}^1P_{2org}$, ${}^2P_{3org}$, ${}^3P_{4org}$, and ${}^1_4T$

[7 Marks]

b) Determine the position and orientation of the object on the work table {O} with respect to reference co-ordinate frame {W}.

[2 Marks]

c) Calculate the position and orientation of the object {O} as it is seen by the robot gripper {4}.

[6 Marks]

Q3. Figure Q3 shows link coordinate frames {i-1} and {i} of a serial link manipulator.

a) Derive an expression for the link transformation matrix ${}^{i-1}_{i}T$ in terms of basic rotation matrices and basic translation matrices.

[2.0 Marks]

b) The DH link-joint parameters of the first three links of PUMA 560 robot manipulator are given in the table Q3. Write expressions for ${}^{0}_{1}T$, ${}^{1}_{2}T$, and ${}^{2}_{3}T$ in terms of basic rotation matrices and basic translation matrices (Note: Specify respective angles, axes of rotation, translations and the directions of translations. However, it is not required to compute matrix elements or to multiply matrices).

[5.0 Marks]

c) Using the template for ${}^{i-1}_{i}T$, calculate the homogeneous transformation matrices ${}^{0}_{1}T$, ${}^{1}_{2}T$, ${}^{2}_{3}T$, and ${}^{0}_{3}T$ when PUMA 560 robot is at $[90^0, 0^0, 60^0]$ arm configuration.

NOTE: The template for ${}^{i-1}_{i}T$ is

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

[5.0 Marks]

d) Determine the origin of the frame {3} with respect to frame {1}.

[3.0 Marks]

Q4. Figure Q4(a) shows a two link robot manipulator having rotational joints with link lengths of $L1$, $L2$.

a) Calculate the velocity of the tip of the arm as a function of joint rates. Give the answer in two forms,

I. With respect to frame {3}
II. With respect to frame {0}

[6.0 Marks]

b) Find the singularities of the two link arm shown in Figure Q4(a) using the Jacobians of joint velocities. Give a physical explanation for the found singularities?

[4.0 Marks]

c) Consider the robot arm shown in Figure Q4 (a) as it is moving its end- effector along the $\hat{X}$-axis at 1.0 m/s, as given in Figure Q4(c). Calculate the joint velocities and show that the joint rates tend to infinity when a singularity is approached.

[5.0 Marks]

Table Q3: DH link-joint parameters of the first three links of PUMA 560 robo manipulator

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	431.8	149.1	θ_3

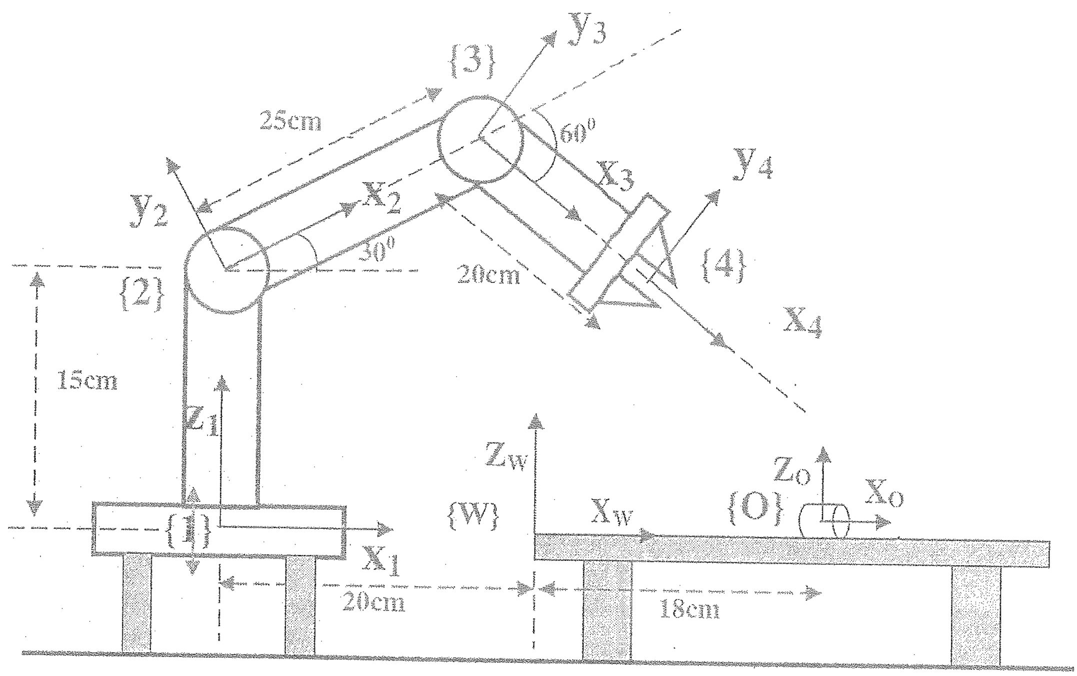

Figure Q2: Robot manufacturing work cell

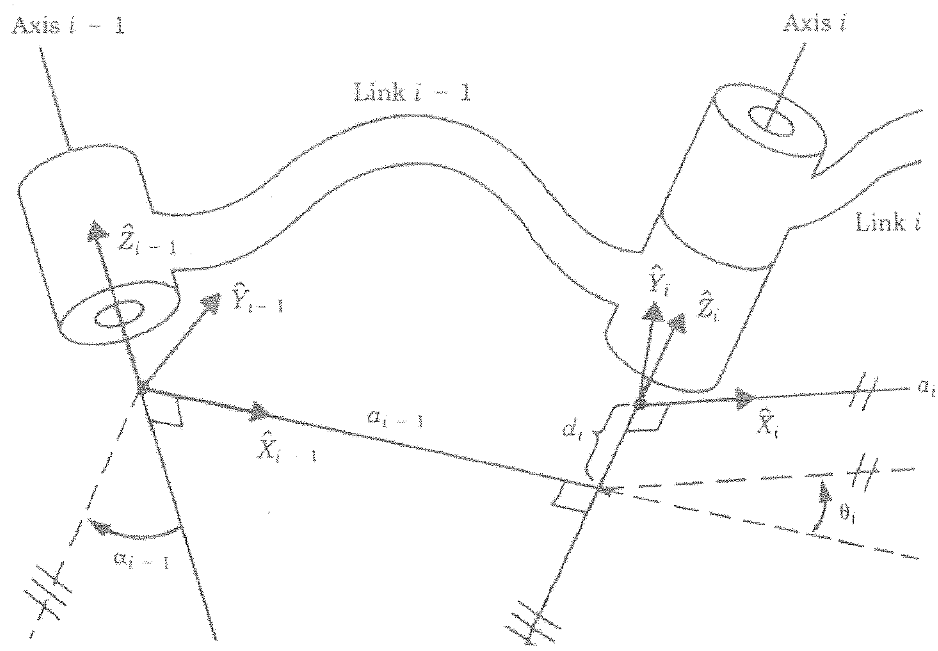

Figure Q3: Co-ordinate frame {i-1} and {i} of a serial link manipulator

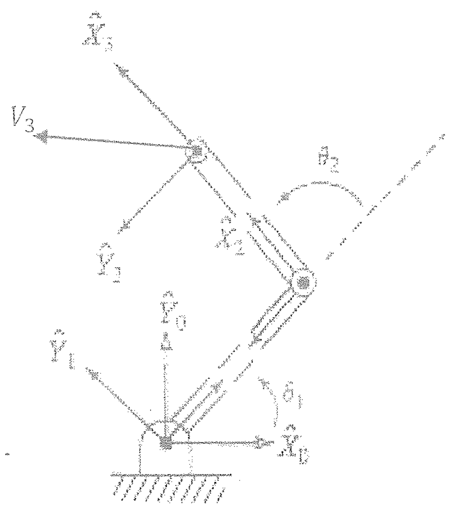

Figure Q4(a): A two link manipulator

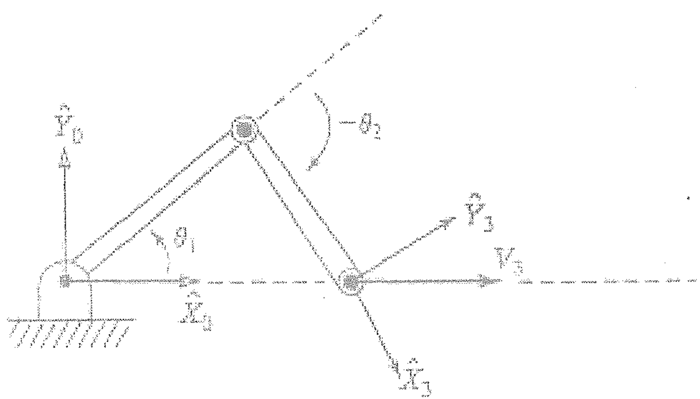

Figure Q4(c): Two link manipulator moving its tip at a constant linear velocity