UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: July 2016

Module Number: EE3205 **Module Name: Signals and Systems**

[Three Hours]

[Answer **all questions,** each question carries **10 marks]**

Q1 a) State expressions to determine the energy and the power of a discrete-time signal.

[2 Marks]

b) Explain whether each of the following statements are true or false.

i) Every bounded periodic signal is a power signal.

ii) If an energy signal $x(t)$ has energy E, then the energy of $x(at)$ is E/a. Assume a is real and positive.

[3 Marks]

c) Determine whether the system described by $y(t) = \sin[x(t+2)]$ is

i) memoryless ii) causal

iii) linear iv) time-invariant

v) stable

[5 Marks]

Q2 a) Write expressions to evaluate the convolution of two discrete-time signals and continuous-time signals respectively.

[2 Marks]

b) Consider two continuous-time signals

$$x(t) = \begin{cases} 2 & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$h(t) = 3e^{-t}$$

Show that $y(t) = x(t) * h(t) = h(t) * x(t)$.

[6 Marks]

c) Explain how the correlation property can be used to identify two transmitted binary bits 0 and 1 at the receiver in a digital communication system.

[2 Marks]

Q3 a) Let $f(t)$ be a function of period $T = 4$ which is defined as on the interval $(-2, 2)$ by,

$$f(t) = \begin{cases} 0 & -2 < t < 0 \\ 2 - t & 0 < t < 2 \end{cases}$$

Determine the trigonometric Fourier series of $f(t)$.

[4 Marks]

b) Consider the following periodic signal.

$$f(t) = \begin{cases} 2 & -\pi \le t \le 0 \\ 0 & 0 \le t \le \pi \end{cases}$$

i) Draw the time domain waveform of $f(t)$ for $-2\pi \le t \le 2\pi$.

ii) Determine the trigonometric Fourier series of the periodic signal $f(t)$.

[4 Marks]

c) Determine the complex exponential Fourier series representation for each of the following signals,

i) $x(t) = \cos(\omega_0 t)$

ii) $x(t) = \sin(\omega_0 t) + \cos(\omega_0 t)$

[2 Marks]

Q4 a) Consider the following rectangular pulse signal.

$$x(t) = \begin{cases} 1 & |t| \le a \\ 0 & |t| > a \end{cases}$$

i) Determine the Fourier transform of the signal $x(t)$.

ii) Draw the magnitude spectrum of the Fourier transform of $x(t)$.

iii) An ideal low-pass filter that has the frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| \le \frac{\pi}{2} \\ 0 & |\omega| > \frac{\pi}{2} \end{cases}$$

If the input to this filter is the signal $x(t)$ with $a = 2$, sketch the shape of the output waveform of the filter. Assume that the filter has unity gain and no phase distortion occurs when the signal passes through the filter.

[5 Marks]

b) Consider the following signal.

$$f(t) = e^{-a|t|}; \quad a > 0$$

i) Draw the time domain waveform of $f(t)$.

ii) Determine the Fourier transform of $f(t)$.

iii) Hence determine the Fourier transform of the following signal.

$$k(t) = \frac{1}{a^2 + t^2}$$

Hint: Use the duality property of the Fourier transform.

iv) Draw the magnitude spectrum of the Fourier transform of $k(t)$.

[5 Marks]

Q5 a) Briefly explain why discrete-time signal processing of continuous-time signals are required.

[2 Marks]

b) Explain why lowpass filtering is needed before obtaining samples of a continuous-time signal.

[2 Marks]

c) Figure Q5 show the frequency spectrum $X(\omega)$ of a bandpass signal $x(t)$.

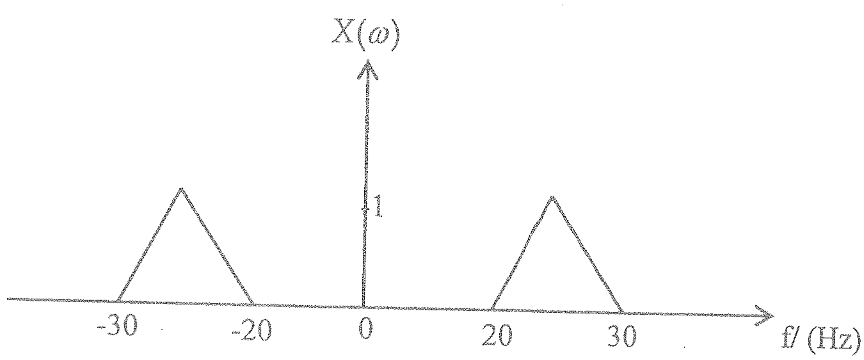

Figure Q5.

i) Draw the spectrum of the signal sampled at a rate of 60 Hz and show that $x(t)$ can be reconstructed from these samples.

ii) A student looks at $X(\omega)$ and concludes that its bandwidth is 10 Hz. Then he decides that a sampling rate of 20 Hz is adequate for sampling $x(t)$. Is it possible to reconstruct $x(t)$ from these samples?

[6 Marks]