



# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 5 Examination in Engineering: July 2016

Module Number: ME5301

Module Name: Computer Aided Design

[Three Hours]

[Answer all questions, each question carries 12 marks]

- Q1. a) What is the relationship between Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM)? [1.0 Mark]
- b) What are the types of Geometric Models? Briefly explain each of them. [3.0 Marks]
- c) List down five major feature types used in the Feature Based Design? [1.0 Mark]
- d) Derive the transformation matrix for rotation of a point in the xy plane around z-axis and translation of a point in the xy plane. [3.0 Marks]
- e) Consider the triangle which is having the vertices of  $(\sqrt{2}, \sqrt{2})$ ,  $(2\sqrt{2}, \sqrt{2})$ ,  $(3\sqrt{2}, 4\sqrt{2})$ . Find the new coordinates when it is subjected to the following transformations.
- Triangle is given a translation of " $r_x = \sqrt{2}$ " along the X axis and " $r_y = \sqrt{2}$ " along the Y axis.
  - Then it is given a rotation of  $45^\circ$  parallel to the Z axis through the vertex  $(2\sqrt{2}, 2\sqrt{2})$ .
- [4.0 Marks]
- Q2. a) Explain the two methods for constructing a single curve. [4.0 Marks]
- b) State the method/methods of construction (mentioned in Q2.a) of the following curves,
- Parametric cubic curve
  - Bezier curve
  - B-Spline curve
- [4.0 Marks]
- c) (i) Prove that a Bezier curve with 4 control points can be described by  $p = UMP$   
Where  $U = [u^3 \ u^2 \ u \ 1]$   
 $P = [P_0 \ P_1 \ P_2 \ P_3]$  and
- $$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$
- (ii) Using the above relationship  $p = UMP$ , construct the Bezier curve whose control points are described by  $(0,0)$ ,  $(60,100)$ ,  $(100,160)$  and  $(160,0)$ .  
(Use the graph sheet to plot the curve.) [4.0 Marks]

- Q3. a) Figure 3 (a) shows the control points for a Bezier curve. Sketch the curve. Draw the line tangent to the curve at the point  $u = 0.5$ . [3.0 Marks]
- b) Figure 3 (b) shows the control points for a B-spline curve of order three (degree two). Sketch the curve. [3.0 Marks]
- c) Find the algebraic coefficients of a PC curve having the following properties
- It lies in the XY plane
  - When  $u=0$   $x=0$  and  $y=0$  and  $\frac{dy}{dx} = \infty$
  - When  $u=0.25$   $x = 1$   $y = 2$  and  $\frac{dy}{dx} = 1$
  - When  $u=1$   $x=4$  and  $y=4$
- [6.0 Marks]

- Q4. a) Show that the element stiffness matrix (in global co-ordinates) of a spring element located at an angle  $\theta$  to the horizontal is  $k$   $\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$ . [4.0 Marks]

- b) Calculate nodal displacements and element stresses for the members of the truss shown in Figure Q4. Cross sectional area of each member =  $500\text{mm}^2$ ,  $E=200\text{GPa}$  and  $P=25\text{kN}$  [8.0 Marks]

- Q5. a) Describe the steps involved in Finite Element Method (FEM) in stress analysis.
- b) With the usual notation, show that the stiffness matrix of a Constant Strain Triangular (CST) element is given by,

$$k = \int_v B^T DB dV \quad [3.0 \text{ Marks}]$$

- c) The CST element of thickness  $t$  mm, shown in Figure Q5, is under the plane stress condition. It has a Young's modulus of  $E$  GPa, specific weight  $\gamma$   $\text{kN/m}^3$  and zero Poisson's ratio. Considering plane stress conditions, find the;

I. Element stiffness matrix for this element.

II. Equivalent load vector

The following are given.

The elasticity matrix for plane stress condition,  $[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$

Constant values  $a_1 = x_2y_3 - x_3y_2$ ,  $a_2 = x_3y_1 - x_1y_3$ ,  $a_3 = x_1y_2 - x_2y_1$ ,  $b_1 = y_2 - y_3$ ,  $b_2 = y_3 - y_1$ ,  $b_3 = y_1 - y_2$ ,  $c_1 = x_3 - x_2$ ,  $c_2 = x_1 - x_3$ ,  $c_3 = x_2 - x_1$  with usual notation.

[7.0 Marks]

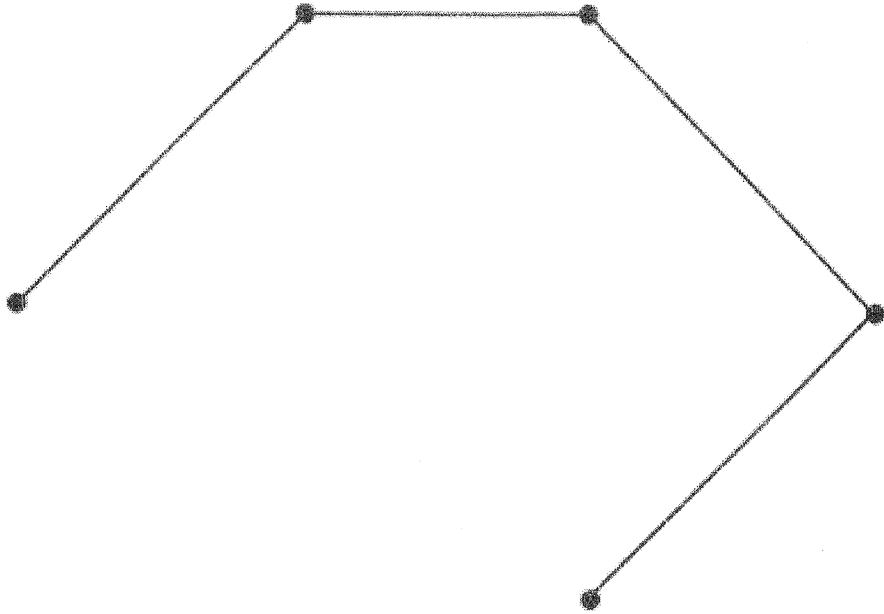


Figure 3 (a) (attach this to your answer booklet)

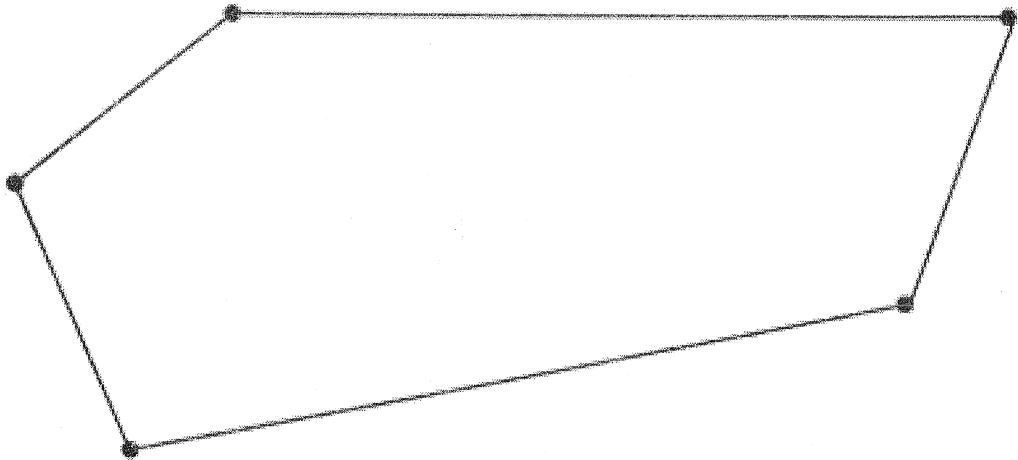


Figure 3 (b) (attach this to your answer booklet)

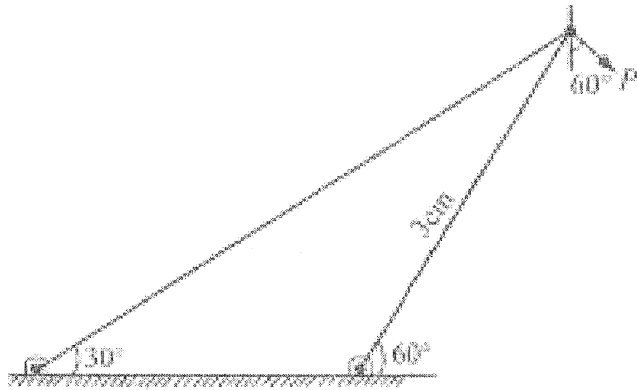


Figure Q4

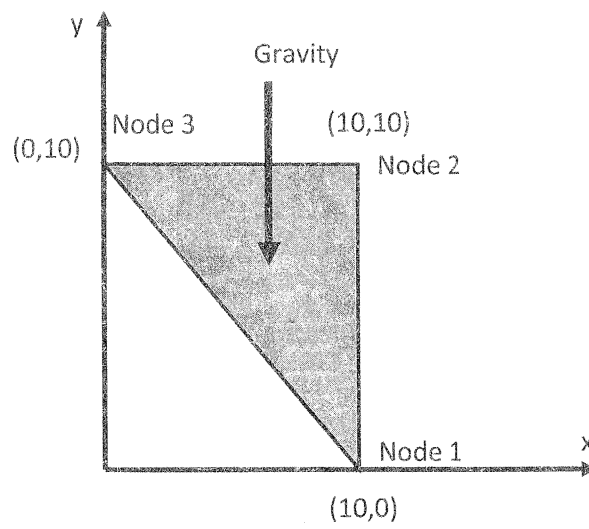


Figure Q5