

Answer Six(06) questions only
 Answer at least 01 question from Part B

All symbols have their usual meaning
 [Acceleration of gravity, $g = 9.8 \text{ ms}^{-2}$, speed of sound, $v = 343 \text{ ms}^{-1}$]

PART A

1. Discuss, briefly, the validity of Newton's second law of motion in inertial and non-inertial frames of reference.

(a) Consider a rocket (Fig. 1) at a launching pad just before taking off from the ground and an astronaut weighing his weight from a step on balance. The weights of the astronaut and the balance are 80 kg and 5 kg, respectively.



Fig. 1

(i) Clearly draw all forces acting on the balance and on the astronaut and calculate them. Clearly indicate action-reaction pairs. What is the magnitude of the force acting on the astronaut by the balance?

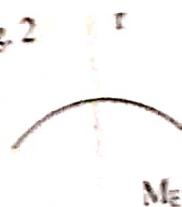
(b) Consider the instant when the rocket takes off with an acceleration of $g/2$.

(i) Calculate all forces and pseudo-forces acting on the balance and on the astronaut as determined by the astronaut.
 (ii) Calculate the reading on the balance at this instant in units of kg?



Fig. 2

(c) Now consider the rocket moving in a circular orbit of radius r around the earth (Mass = M_E) at a constant speed v as shown in figure 2.



(i) Draw and label all forces acting on the rocket as determined by an observer on the earth.
 (ii) Find all forces and pseudo-forces acting on the balance and the astronaut as determined by the astronaut.
 (iii) Calculate the reading on the balance at this instant in units of kg?

2. Explain the Newton's experimental law for a head-on collision of two objects of masses m_1 and m_2 and speeds u_1 and u_2 , respectively. Assume that u_1 and u_2 are in the same direction

and $u_1 > u_2$. Let v_1 and v_2 be the speeds after the collision. How would you apply this law for a perfectly elastic collision?

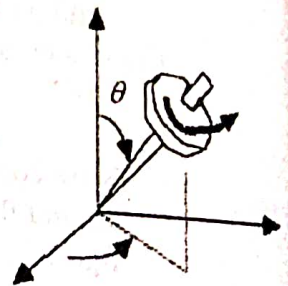
- Write an expression using conservation of linear momentum for the perfectly elastic collision of the above two objects. Discuss, briefly, the validity of this relationship for inelastic collisions.
- If the collision is perfectly elastic, obtain expressions for v_1 and v_2 in terms of given quantities.
- Consider a meteorite of mass one tenth of the mass of earth (M_E) and speed four times the speed of the earth (u_E) making a head-on collision with the earth. Assume that the speeds of both objects were in the same direction at the instant of collision. If the two objects move as a single object after the collision, obtain an expression for the common speed just after the collision.

3. The earth is rotating about its axis with the angular speed ω . Explain all the forces necessary to describe the motion of a particle of mass m and speed v by an observer on the earth.

- A low pressure region is developed in the atmosphere of the southern hemisphere of the earth. Explain, with reasons, the development of wind patterns around the low pressure region.
- Consider a coordinate system xyz of the earth. The angular velocity of the earth is $\vec{\omega} = 4\hat{k}$ relative to an inertial coordinate system S . The position of a particle on the earth at time t with respect to frame xyz is given by $\vec{r} = (2t^2 + t)\hat{i}$. Calculate,
 - apparent velocity of the particle.
 - true velocity of the particle.

4. What is meant by precession? Explain, with reasons, the precession of a spinning top.

A spinning top of mass M , moment of inertia I about the symmetry axis and angular speed ω is shown in the figure. The center of gravity of the top is located at a height R along the axis.



- Derive an expression for the rate of precession (Ω_p) of the top.
- Calculate Ω_p when it spins at the rate of 4 rev/s. Indicate the direction of precession. ($M = 0.2$ kg, $I = 4 \times 10^{-2}$ kg m², and $R = 6$ cm)
- Find the direction and magnitude of the horizontal force that has to be exerted by the floor to maintain the stability of the top. Take $\theta = 30^\circ$.
- The earth spins about its axis. Could it under go precession? Explain, briefly.

Write down a condition that must be satisfied by a conservative force. Prove that any central force is a conservative force. State four examples, two each for conservative and non-conservative forces.

- (a) Which of the following forces are conservative? Prove your answer.
 (i) $y^4 \hat{i} + 4y^3 x \hat{j}$ (ii) $yx \hat{i} + xy \hat{j}$

- (b) The potential of a diatomic molecule can be written as

$$U(r) = U_0 \left[\left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^6 \right] \text{ where } r_0 \text{ and } U_0 \text{ are constants.}$$

- (i) Find the force F_r .
 (ii) Find the value of r at the minimum of the potential.

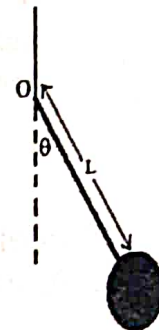
State the Gauss' theorem for gravitational fields.

The density of the earth is given by ρ_1 and ρ_2 in the regions $r < a$ and $a < r < b$ respectively, where b is the radius of the earth.

- (a) Obtain an expression for the gravitational field intensity at a distance r from the center for each of the following cases.
 (i) $r < a$ (ii) $a < r < b$ (iii) $r > b$
- (b) Derive an expression for the gravitational potential $V(r)$ at a distance $r (> b)$ from the center of the earth.

PART B

A uniform disk of radius R and mass M is attached to the end of a uniform rigid rod of mass m and length L as shown in the figure. The system is pivoted from the point O. Neglect any friction at pivot point and possible air resistance.



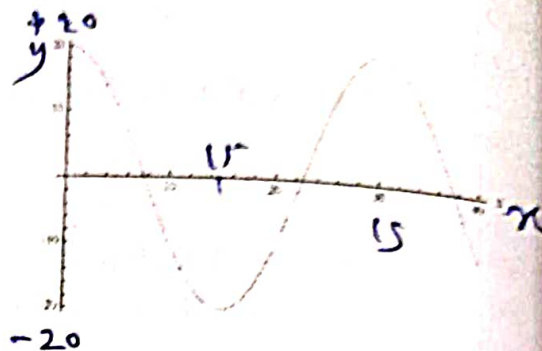
- (a) Show that the moment of inertia of a rigid rod of mass m and length L about an axis perpendicular to the plane of the rod passing through one end of the rod is $\frac{1}{3}mL^2$.
- (b) If the moment of inertia of the disk with mass M about an axis passing through the center of mass perpendicular to the plane of the disk is $\frac{1}{2}MR^2$, find the moment of inertia perpendicular to the plane of the rod plus disk system about the pivot point O.
- (c) If the system is displaced by a small angle θ in a vertical plane as shown in the figure, write down the equation of motion for oscillations.
- (d) Show that the period of oscillation T for the above system is given by

$$T = 2\pi \sqrt{\frac{\left[\frac{1}{3}m + M \right] L^2 + \frac{3}{2}MR^2 + 2MLR}{g \left[MR + \left(\frac{m}{2} + M \right) L \right]}}$$

8. A sinusoidal wave traveling in the positive x direction has an amplitude of 20 m, wavelength of 30 m and frequency of 8 Hz. A snapshot of the wave at $t=0$ is shown in the figure.

(a) Calculate the following quantities.

- (i) Wave number, k
- (ii) Period, T
- (iii) Angular frequency, ω
- (iv) Wave speed, v
- (v) Phase constant, ϕ



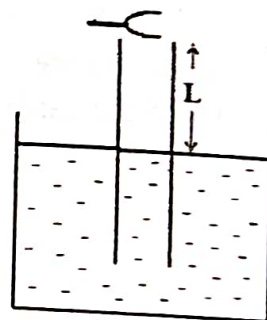
Write down a general equation for the wave.

(b) A vertical pipe, open at both ends, is submerged in water as shown in the figure. A tuning fork vibrating with an unknown frequency is placed near the end of the pipe. The length of the air column can be adjusted by moving the pipe vertically while keeping the bottom end submerged in water. The smallest value of L for which a peak occurs in the sound intensity is 10 cm.

(i) Draw the corresponding diagrams for this harmonic and find the frequency of the tuning fork.

(ii) Draw the diagrams for next two resonant conditions and find the corresponding values of L .

(iii) When the wind blows through a cylindrical pipe of length L , which is open at both ends, it makes a howling noise. Determine the frequencies of the first three harmonics of the pipe.



9.

(a) A bell sounding alarm of 600 Hz is dropped from a place of 15 m high above the ground by you. Find the frequency of the alarm you hear just before it hits the ground.

(b) A submarine traveling towards north under water at a speed of 8 ms^{-1} emits a sonar at frequency of 1400 Hz. The speed of sound in water is 1533 ms^{-1} . A whale is swimming towards south along the same line towards the submarine with a speed of 9 ms^{-1} .

(i) What is the frequency of the sonar detected by the whale?

(ii) As the whale and the submarine are approaching each other, some of the sonar waves sent from the submarine reflects from the whale's body and returns to the submarine again. What is the frequency of these reflected waves detected by the submarine?

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