UNIVERSITY OF RUHUNA

B.Sc. General Degree Level II (Semester II) Examination – January/February 2018

Subject: PHYSICS
Course Unit: PHY2214

Time: 02 hours & 30 minutes

Part II

Answer FIVE (05) Questions only.

(All symbols have their usual meaning)

$h = 6.626 \times 10^{-34}$ Js

$\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

$\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$

$e = 1.6 \times 10^{-19} C$

$c = 3 \times 10^8 ms^{-1}$

$m_e = 9.1 \times 10^{-31}$ kg

R (Rydberg's constant) = $1.097 \times 10^7 m^{-1}$

1 u = 931.5 MeV

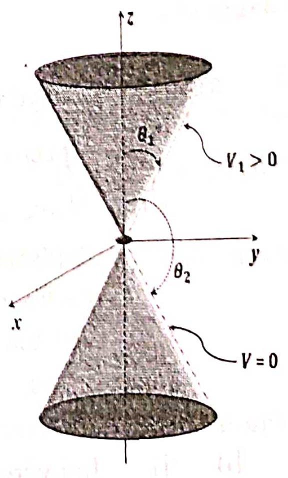
In spherical coordinates,

Laplace's equation: $$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2} = 0$$

Del operator: $$\vec{\nabla} = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\hat{\varphi}$$

1. a) Starting from the differential form of the Gauss' law, obtain Poisson's and Laplace's equations. **(05 marks)**

b) Two conducting cones have been put together at their vertices as shown in the figure. The vertices are insulated at the origin (at $\vec{r} = 0$). The upper cone has a constant potential of $V_1 (> 0)$ at an angle θ_1 from the z-axis and the lower cone has a zero potential at an angle θ_2 from the z-axis.



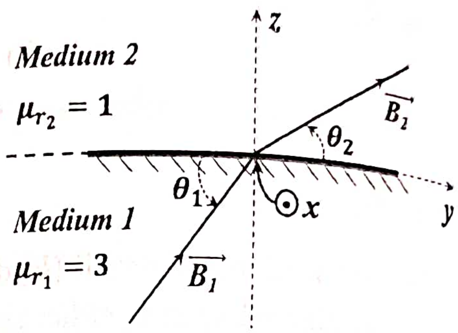
(i) Solve the Laplace's equation to find the potential at a point in the outer region between the cones.

{Hint: $\int \frac{1}{\sin\theta} d\theta = \ln\left|\tan\frac{\theta}{2}\right| + constant$} **(09 marks)**

(ii) Derive an expression for the electric field intensity $\vec{E}$ at that point. (05 marks)

(iii) Assuming $\theta_1 = 30^0$, $\theta_2 = 150^0$ and $V_1 = 100$ V, calculate the potential and the electric field intensity at a distance of 1 m from their vertices on the *horizontal plane* (x-y plane). (06 marks)

2. a) Two magnetic media are separated by a *current free interface* on the *x-y* plane as shown in the given figure. The magnetic field vector in the *medium 1* is given by $\overrightarrow{B_1} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$. Relative permeabilities of *medium 1* and *medium 2* are $\mu_{r_1} = 3$ and $\mu_{r_2} = 1$, respectively.



(i) Calculate $\overrightarrow{H_1}$, $\overrightarrow{H_2}$ and $\overrightarrow{B_2}$. (10 marks)

(ii) Find the angles θ_1 and θ_2 between the field vectors and a tangent to the interface. (04 marks)

b) (i) Write down Maxwell's equations in electrodynamics for a free space. (04 marks)

(ii) Electric field intensity vector in *free space* is given by $\vec{E} = E_0 \cos(\omega t - \beta z)\,\hat{\imath}$. Obtain expressions for $\vec{D}$, $\vec{B}$ and $\vec{H}$ in free space. (07 marks)

3. a) (i) A hydrogen atom has only one electron, but many spectral lines are formed in the spectrum of hydrogen. Explain the above statement. (04 marks)

(ii) Name the different spectral line series of hydrogen spectrum. Write down an expression for λ in terms of R (Rydberg's constant) and n for the spectral lines of each series. (06 marks)

(iii) Calculate the wavelength of the first member and the limiting member of Balmer series. (05 marks)

b) (i) In hydrogen atom, the electron is in the third orbit. A photon of energy 10.04 eV knocks out the electron. Calculate the energy of ejected electron. Ionization energy of H atom is 13.6 eV. (05 marks)

(ii) What is the wavelength of radiation emitted when the electron jumps from the second excited state to the ground state?

(05 marks)

4. Natural uranium must be processed to produce uranium enriched in ^{235}U for bombs and power plants. The processing yields a large quantity of nearly pure ^{238}U as a by-product, called "depleted uranium". Because of its high mass density, it is used in armor-piercing artillery shells.

The density of uranium is $18.7 \times 10^3\ kgm^{-3}$.

($^{238}_{92}U = 238.050783\ u$, $^{4}_{2}He = 4.002603\ u$, $^{206}_{82}Pb = 205.974449\ u$ and $^{0}_{-1}e = 0.0005485\ u$)

a) Find the length of the side of a 70.0 kg cube of ^{238}U.

(04 marks)

b) The isotope ^{238}U has a long half-life of $4.47 \times 10^9\ yr$. As soon as one nucleus decays, it begins a relatively rapid series of steps that together constitute the net reaction $^{238}_{92}U \rightarrow 8(^{4}_{2}He) + 6(^{0}_{-1}e) + ^{206}_{82}Pb + Q_{net}$. Calculate the net energy released in the decay process (Q_{net}).

(04 marks)

c) Argue and show that a radioactive sample with decay rate R and decay energy Q has power output $P = QR$.

(04 marks)

d) Consider an artillery shell with a jacket of 70.0 kg of ^{238}U. Find its power output due to the radioactivity of the uranium and its daughters. Assume that the shell is old enough that the daughters have reached steady-state values. Express the power in joules per year.

(09 marks)

e) A 17-year-old soldier of mass 70.0 kg works in an arsenal where many such artillery shells are stored. Assume that his radiation exposure is limited to absorbing 45.5 mJ per year per kilogram of body mass. Find the net rate at which he can absorb energy of radiation, in joules per year.

(04 marks)

5. a) The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is N_0, the number N at time t is given by

$N = N_0 e^{-t/T}$, where $T = 2.20\ \mu s$ is the mean lifetime of the muon. Suppose the muons move at speed 0.95c with respect to an observer.

(i) What is the observed lifetime of the muons by the observer? **(05 marks)**

(ii) How many muons remain after traveling a distance of 3.0 km in the rest frame of the muons?

(06 marks)

b) (i) Consider a particle traveling in the x-direction at velocity u with respect to an inertial reference frame S. Frame S' moves along the x-axis at velocity v with respect to S.

Write down velocity transformation equations.

(06 marks)

(ii) A stationary observer on Earth observes spaceships A and B, which are moving in the same direction towards the Earth. Spaceship A has speed 0.5c and spaceship B has speed 0.8c. Calculate the velocity of spaceship A as measured by an observer at rest in spaceship B.

(08 marks)

6. a) The total energy of a particle with speed u is given by γmc^2, where $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$, and m is the rest mass of the particle. This total energy of the particle is the sum of the particle's kinetic energy and the rest energy, mc^2. The relativistic momentum of the particle is γmu.

(i) Write a simple expression for the kinetic energy of the particle in terms of the given quantities.

(04 marks)

(ii) How fast does the particle need to move for its kinetic energy to account for 2/3 of the total energy?

(05 marks)

(iii) What is the momentum of the particle when it is moving at the speed calculated in a) (ii) ?

(04 marks)

b) The hare and the tortoise decide to have another race. They start from the same point and race in the same direction, but the hare decides to give the tortoise a chance by letting him race only half the distance as shown in the figure below. The race takes place and the referee, who is at rest with respect to the ground, finds that both animals cross their respective finish lines at the *same time*. The trajectories of the hare and the tortoise are shown on the space-time diagram. Answer the following questions.

(i) At which of the points labeled A through G in the space-time diagram does the hare cross its finish line?

(03 marks)

(ii) At which point does the tortoise cross its finish line?

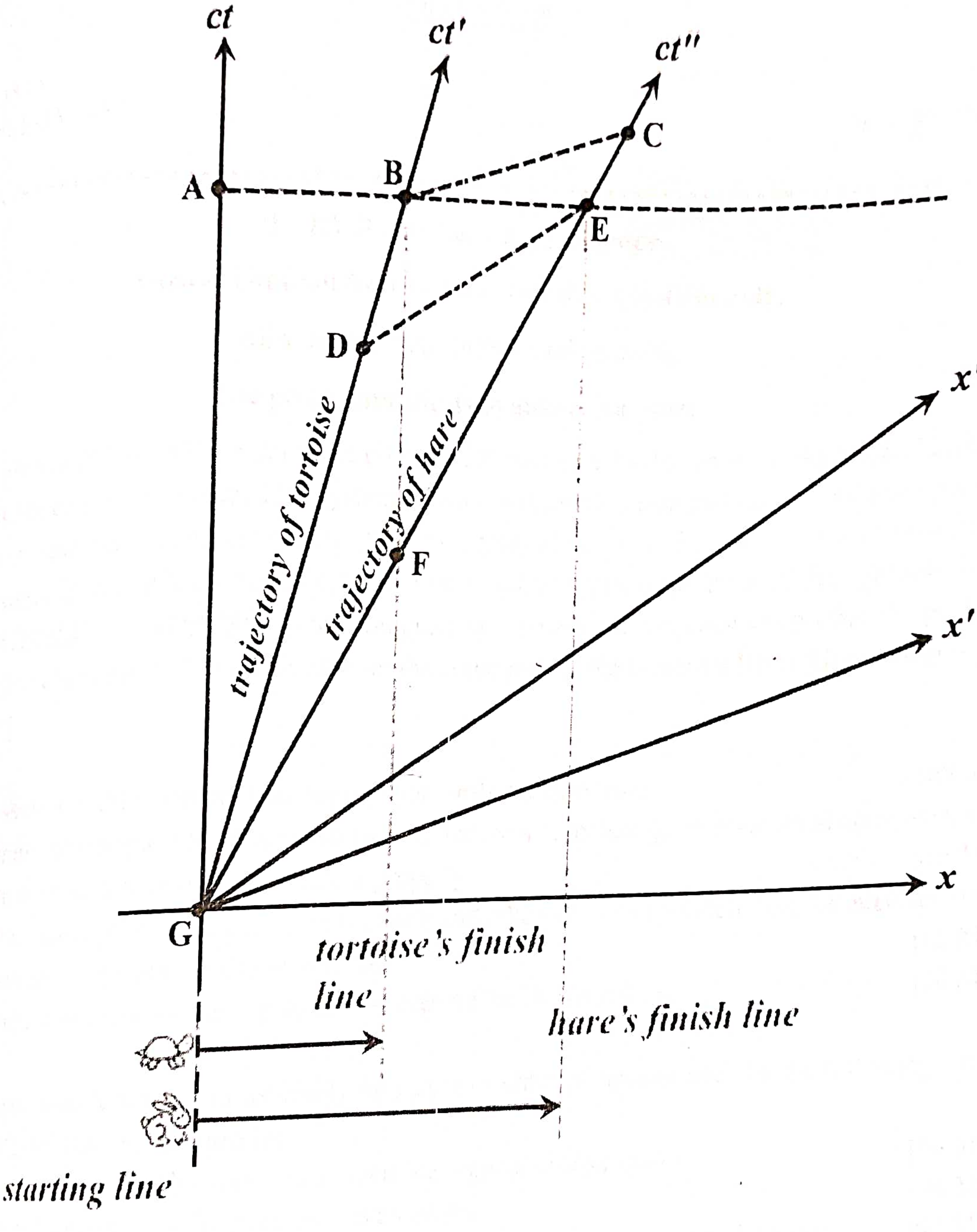
(03 marks)

(iii) At which point is the tortoise *when* the hare crosses its finish line in the *hare's frame*?

(03 marks)

(iv) At which point is the hare *when* the tortoise crosses its finish line in the *tortoise frame*?

(03 marks)


ct
ct'
ct''
C
A
B
E
D
x''
trajectory of tortoise
trajectory of hare
F
x'
x
G
tortoise's finish line
hare's finish line
starting line

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