UNIVERSITY OF RUHUNA

BACHELOR OF SCIENCE (GENERAL) DEGREE LEVEL III (SEMESTER I)

EXAMINATION – JUNE/JULY 2015

Subject: PHYSICS
Course Unit: PHY3114

Time: 02 hours & 30 minutes

Part II

Answer at least 01 (ONE) question from each of the parts A, B and C.

Answer FIVE (05) Questions only.

(All symbols have their usual meanings.)

Planck's constant, $h = 6.626 \times 10^{-34}$ Js
Avogadro's number, $N_A = 6.022 \times 10^{23}$
Charge of an electron, $e = 1.6 \times 10^{-19}$ C
Boltzmann constant, $k = 1.38 \times 10^{-23}\, JK^{-1}$
Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

$$\int_0^\infty x^3 e^{-\alpha x^2}\,dx = \frac{1}{2\alpha^2}$$

PART A

1. (a) What is meant by the primitive lattice translational vectors? Hence, define reciprocal lattice vectors of the primitive lattice.

   (b) (i) What are the primitive lattice translational vectors of a FCC structure?
   (ii) Find the reciprocal lattice vectors of the FCC structure.
   (iii) Find the volume of the primitive cell in the reciprocal lattice.
   (iv) Find the reciprocal lattice vector, G.

2. (a) Write down the Bragg's law of X-ray diffraction in crystals. Define all terms.
   (b) State under what conditions the Bragg's law is derived.
   (c) In addition to X-ray diffraction, what other diffraction methods are used to study crystals structures? Discuss them briefly.
   (d) X-ray of wavelength 0.86 Å is diffracted by a cubic KCl crystal of density $1.99 \times 10^3 kgm^{-3}$. Calculate the interplaner spacing for (0 2 0) planes and glancing angle for the second order Bragg reflection from these planes. The molecular weight of KCl is 74.6 g. The number of atoms in the KCl unit cell is 4.

3. (a) (i) What is meant by density of states for a free electron gas?

(ii) Obtain an expression for the density of states for a free electron gas in a one dimensional box of length L. Hence, obtain the relationship between density of states and energy E_n. ($E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$)

(b) (i) What is meant by mobility?

(ii) Write down the expression for the conductivity of an intrinsic semiconductor using carrier mobilities and other relevant quantities.

(iii) Optical excitation of intrinsic germanium creates an average density of 10^{18} conduction electrons per m^3 in the material at liquid nitrogen temperature. At this temperature, the electron and hole mobilities are equal, $\mu = 0.5\ m^2/Vs$. If a voltage of 100 V is applied across a germanium cube of side length 0.01 m under these conditions, calculate the current through the cube.

PART B

4. (a) State canonical distribution and partition function, by defining all the terms.

(b) Show that the mean energy $(\overline{E})$ of a system in equilibrium at absolute temperature T is given by $\overline{E} = -\frac{\partial(\ln Z)}{\partial\beta}$.

(c) Consider a system of three particles, each having a magnetic moment μ_0, placed in an external magnetic field B.

(i) What are the possible energy states of the system in equilibrium at absolute temperature T?

(ii) Write the partition function of the system.

(iii) Obtain an expression for the probability of finding the system is in equilibrium with total energy $-\mu_0 B$.

(iv) Find the mean total energy of the system.

5. In Maxwell's speed distribution, the mean number of molecules per unit volume with speed in the range between v and $v+dv$ is given by $F(v)dv = 4\pi n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv$.

(a) By using above distribution show that the mean speed of a molecule is equal to $\sqrt{\frac{8kT}{\pi m}}$.

(b) Show that the most probable speed (v_m) is equal to $\sqrt{\frac{2kT}{m}}$.

(c) Determine the average speed of an O_2 molecule at room temperature of 27°C. (Mass of O_2 molecule is 5.31×10^{-26} kg.)

(d) Assuming n = 2.45×10^{21}, calculate the number of O_2 molecules having speeds in the range v and $v+dv$ and in the vicinity of most probable speed, v_m (assume $dv = 10^{-6} v_m$).

6. Consider a gas of N identical particles in volume V in equilibrium at temperature T. Mean number of particles in a single particle state is given by $\bar{n}_s = -\frac{1}{\beta}\frac{\partial \ln Z}{\partial \varepsilon_s}$. ($Z$ is the partition function of the gas. ε_s is the energy of the particle in state s.)

(a) By using above expression find the dispersion of n_s and show that it is equal to $-\frac{1}{\beta}\frac{\partial(\bar{n}_s)}{\partial \varepsilon_s}$.

(b) Compare and contrast the statistics obeyed by fermions and bosons.

(c) Consider a gas consists of two particles A and B. Assume that each particle can be in one of three possible quantum states, s = 1, 2 and 3. Describe all possible arrangements of these two particles in the three states according to Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac statistics, in tabular form.

(d) Show all possible ways of arranging four identical particles in two single particle states according to Fermi-Dirac and Bose-Einstein statistics.

PART C

7. (a) Write down the Heisenberg's uncertainty principle and describe its meaning in simple terms.

(b) Write down the momentum operator for a particle travelling in x-direction with momentum p_x.

(c) If $\hat{x}$ and $\hat{y}$ are position operators, obtain commutations $[\hat{x}, \hat{p}_x]$ and $[\hat{y}, \hat{p}_x]$. Explain answers using the uncertainty principle.

(d) Write down one characteristic of a Hermitian operator. Show that $\hat{p}_x$ is a Hermitian operator.

(e) An electron moving with a speed of 2.0×10^6 m/s with an uncertainty of 0.1%. What is the uncertainty of its position? Calculate the wavelength of the electron.

8. A particle is confined to a 2-d infinite potential well as defined below.

$$V = 0,\ 0 < x < l_1 \text{ and } 0 < y < l_2;\ l_2 > l_1$$
$$= \infty,\ \text{outside}$$

(a) Find normalized eigenfunctions and energy eigenvalues for the particle.

(b) Write down eigenvalues and corresponding eigenfunctions for the ground state and first two excited states. Discuss degeneracy of the eigenstates, if any.

(c) Discuss, the orthogonality of eigenfunctions in the first three states.

9. (a) $\hat{A}, \hat{B}$ and $\hat{C}$ are three operators. Prove following commutation relations.

(i) $[\hat{A} + \hat{B}, \hat{C}] = [\hat{A}, \hat{C}] + [\hat{B}, \hat{C}]$

(ii) $[\hat{A}\hat{B}, \hat{C}] = [\hat{A}, \hat{C}]\hat{B} + \hat{A}[\hat{B}, \hat{C}]$

(b) Hence, if $\hat{D}$ is another operator, **expand** (no need to prove) following commutation relations.

(i) $[\hat{A} + \hat{B}, \hat{C} + \hat{D}]$

(ii) $[\hat{A}\hat{B}, \hat{C}\hat{D}]$

(c) $\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$, $\hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z$ and $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$

Using relations obtained in (b) and any other commutation relations that above dynamical operators satisfy, show that $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$.

(d) How do energy levels of the electron in the hydrogen atom vary with the principal quantum number n? The ground state energy is -13.6 eV. What are the energies of first and second excited states?

(e) Sketch possible energy levels up to n=3 and indicate possible values for quantum numbers l and m. Show possible eigenfunctions in each level using notation ψ_{nlm}.

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