

Part II

Answer FIVE (05) Questions only.

(All symbols have their usual meaning)

Planck's constant, $h = 6.626 \times 10^{-34}$ Js
Avogadro's number, $N_A = 6.022 \times 10^{23}$
Speed of light, $c = 3 \times 10^8$ ms⁻¹
eV = 1.602×10^{-19} J

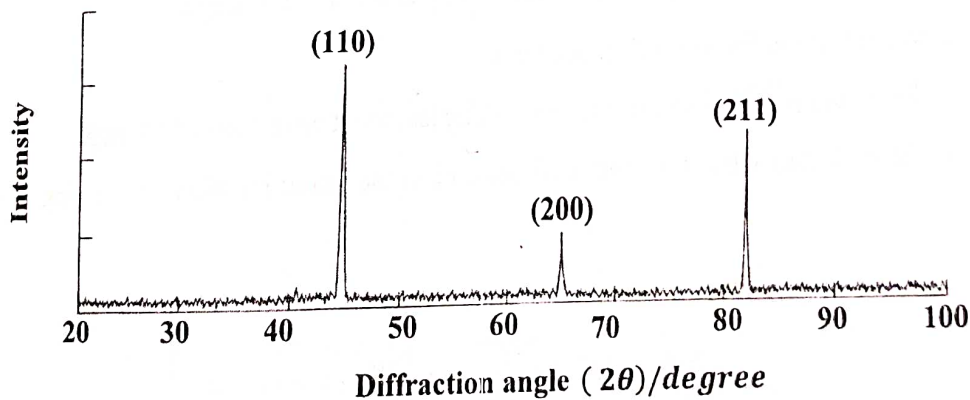
Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ JK⁻¹

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

1 a.m.u = 1.66×10^{-27} kg

PART A

1. The figure below shows first order x-ray diffraction pattern for iron (Fe). Wavelength of the monochromatic x-radiation is 0.154 nm. The three peaks (110), (200) and (211) occurs at diffraction angle 45°, 65° and 82°, respectively



- (a) Compute the interplanar spacing for second set of planes. [05-marks]
(b) Determine the lattice parameter for Fe. [05-marks]
(c) X-ray diffraction reveals that Fe forms cubic unit cell. Calculate the volume and mass of the cell. The density of Fe is 7.83×10^3 kg/m³. [05-marks]
(d) Molecular weight of Fe is 55.85×10^{-3} kg. Determine the type of cubic unit cell whether (SC, FCC or BCC). [05-marks]
(e) Find the radius of a Fe atom. [05-marks]

2.

(a) Sketch the Fermi levels of p-type and n-type semiconductors. [06-marks]

(b)

i) What is an extrinsic semiconductor? [03-marks]

ii) Write down the equation for conductivity of an extrinsic semiconductor, defining all terms. [04-marks]

iii) Obtain an expression for resistivity of an intrinsic semiconductor. [04-marks]

iv) Calculate the resistance of an intrinsic semiconductor rod at room temperature, which is 2 cm long, 1 mm wide and 1 mm thick. Concentration of intrinsic carriers at room temperature is $2 \times 10^{19} \text{ m}^{-3}$ and mobilities of electrons and holes are 0.39 and $0.19 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, respectively. [04-marks]

3. Consider the random walk problem in one-dimension. Assume that the drunk takes steps of equal length L . Probabilities of taking a step to the right and to the left are p and q , respectively. [08-marks]

(a) Write down the probability, $W_N(n_1)$, of taking n_1 steps to the right and n_2 steps to the left in a total of N steps. [05-marks]

(b) Find the mean number of steps taken to right (\bar{n}_1) in a total of N steps. [05-marks]

(c) Obtain an expression for mean displacement. [05-marks]

(d) If $L = 0.25 \text{ m}$ and $p = 0.6$ calculate the mean displacement after total of 60 steps. [05-marks]

(e) What is the probability that the drunk will again be at the initial location after taking 10 steps? [05-marks]

$$\left(\text{Note: } np^n = p \frac{\partial p^n}{\partial p} \text{ and } \sum_{n=0}^N W_N(n) = (p+q)^N \right)$$

4.

(a) State *Canonical distribution*. [04-marks]

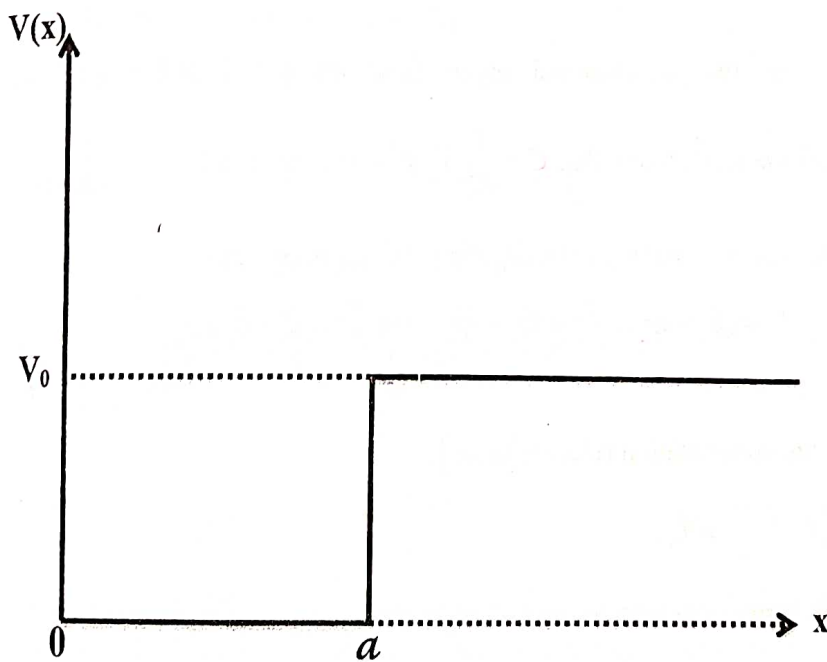
(b) Show that the mean energy (\bar{E}) of an isolated system in equilibrium with a heat reservoir at absolute temperature T is given by $\bar{E} = -\frac{1}{Z} \frac{\partial Z}{\partial \beta}$, where Z is the partition function of the system. [06-marks]

(c) The energy of a one-dimensional simple harmonic oscillator is given by $E_n = \left(n + \frac{1}{2} \right) \hbar \omega$.

Quantum number n can assume the possible integer values $n = 0, 1, 2, 3, \dots, \infty$ and ω is the angular frequency of the oscillator. Suppose that such an oscillator is in thermal contact with a heat reservoir at low temperature T , so that $\frac{kT}{h\omega} \ll 1$.

- i) Find the ratio of, the probability of the oscillator being in the first excited state to the probability of its being in the ground state. [05-marks]
- ii) Write the partition function of the system. [05-marks]
- iii) Assuming that only the *ground state* ($n = 0$) and *first excited state* ($n = 1$) are appreciably occupied, find the mean energy of the oscillator as a function of the temperature T . [05-marks]

5. (a) An electron of mass m is confined to a one-dimensional potential as shown in the figure. Consider that the electron energy $E > V_0$.



- i) Solve the Schrödinger equation for regions, $0 < x < a$ and $x > a$. [06-marks]
- ii) Find physically acceptable solutions for the two regions. [04-marks]
- iii) What is probability of finding the electron in the region $x > a$. [05-marks]

(b). Energy of a particle in one dimensional force free region of length L at quantum state n is given by $E_n = \frac{n^2 h^2}{8mL^2}$. Write down the energies of the electron at the first excited state and the ground state in the bounded region $0 < x < 10 \text{ fm}$. Hence, find the energy and wavelength of the photon emitted when electron undergoes a transition from first excited state to the ground state. In what region of the electromagnetic spectrum does this wavelength belong? [10-marks]

6. (a). Consider two eigen functions ψ_1 and ψ_2 corresponding to eigen values λ_1 and λ_2 . Discuss characteristics of ψ_1 and ψ_2 under following conditions. [04-marks]

- i) $\lambda_1 = \lambda_2$
- ii) $\lambda_1 \neq \lambda_2$

(b). Superposition of two orthonormal eigen functions $\psi_1(x)$ and $\psi_2(x)$ can be written as

$$\psi(x) = C[\psi_1(x) + \psi_2(x)]. \text{ Show that } C = \frac{1}{\sqrt{2}} \text{ if } \psi(x) \text{ is normalized.}$$

[06-marks]

(c). Consider following relationships of a angular momentum operator \hat{L} .

$$\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y, \quad \hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z, \quad \hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x \quad \text{and} \quad \hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

i) Write down the commutation relation $[\hat{x}, \hat{p}_x]$. [02-marks]

ii) Show that $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$. [05-marks]

iii) Show that \hat{L}^2 operator commutes with \hat{L}_x operator. [05-marks]

iv) If $Y(\theta, \phi)$ is an eigen function of \hat{L}^2 , write down the eigen value equation for \hat{L}^2 operator. Hence, define the eigen values of the \hat{L}^2 operator. [03-marks]