UNIVERSITY OF RUHUNA

BACHELOR OF SCIENCE (GENERAL) DEGREE LEVEL III (SEMESTER I) EXAMINATION – NOVEMBER/DECEMBER 2019

Subject: PHYSICS
Course Unit: PHY 3114

Time: 02 hours & 30 minutes

Part II

Answer FIVE (05) Questions only.

(All symbols have their usual meaning)

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Boltzmann constant, $k_B = 1.38\text{x}10^{-23}$ JK^{-1}

Avogadro's number, $N_A = 6.022\text{x}10^{23}$

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3 \times 10^8$ ms^{-1}

1 eV = 1.602×10^{-19} J

1 a.m.u = 1.66×10^{-27} kg

1. Unit cell of copper (Cu) metal is face centered cubic (FCC).

(a) Sketch a FCC unit cell and describe its characteristics. **[05 marks]**

(b) How many atoms are in a single unit cell? **[03 marks]**

(c) Calculate the unit cell dimension (a), given that atomic weight of Cu is 63.54×10^{-3} kgmol^{-1} and its density is 8890 kgm^{-3}. Hence, find the volume of the unit cell. **[06 marks]**

(d) Obtain a relationship between the unit cell dimension (a) and the radius of Cu atom (r). Hence, find r. **[05 marks]**

(e) What is meant by packing fraction? Obtain the value of the packing fraction for Cu. **[06 marks]**

2. Consider the case of N electrons confined to a one-dimensional box of length L. Energy of an electron in the nth level can be written as $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$.

(a) Describe the Fermi level (n_f). How would N electrons be filled up to level n_f? **[04 marks]**

(b) Hence, drive an expression for total energy of N electrons (E_0) in the system. **[08 marks]**

(c) If $L = 1$ Å and $N = 100$, calculate energy and momentum of an electron at the Fermi level. **[08 marks]**

(i) Hence find the total energy of electrons in this system. [0:

(ii) What assumptions did you have to make in this case?

[02

(You may use $\sum_{n=1}^{s} n^2 = 1^2 + 2^2 + \ldots\ldots\ldots + s^2 \approx \frac{s^3}{3}$ when s>>1)

3. Energy levels of a one-dimensional harmonic oscillator, in thermal equilibrium with a heat reservoir at absolute temperature T is given by $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, where $n = 0,1,2,3,\ldots\ldots.$

(a) Write down the partition function (Z) and show that $Z = \dfrac{e^{-\beta\hbar\omega/2}}{1 - e^{-\beta\hbar\omega}}$. [07

(You may use $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, when $x < 1$)

(b) Show that mean energy of the oscillator, $\bar{E}$, can be given as $\bar{E} = \dfrac{\hbar\omega}{2} + \dfrac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$. [07 n

(c) Mean energy of the oscillator can also be written as $\bar{E} = \left(\bar{n} + \frac{1}{2}\right)\hbar\omega$. Where $\bar{n}$ is the mean value of oscillations. Find $\bar{n}$.

[04 m

(d) Find limiting values of $\bar{n}$ at very high and low temperatures. [04 m

(e) Derive an expression for the Helmholtz free energy (F) for this system. [03 m

4.

(a) Describe "effusion", stating conditions/assumptions that must be satisfied for such a process to take place. [06 ma

(i) A sealed cubical box of side $L = 0.015$m is filled with oxygen gas of N molecules kept at 300 K. A face of the box has a small hole of diameter 3μm. The gas inside leaks out slowly through this hole. The mean free path of oxygen molecules inside the container is 10^{-3} m. How long (t) will it take for the gas to decrease to half of its original amount (i.e. $N/2$)? Assume that pressure outside the container is very low and leakage back into the container is negligible. Obtain the necessary expression for t in terms of given parameters.

Note: Mean speed of the oxygen gas molecule inside the container is $445.42\ ms^{-1}$. **[08 marks]**

(b) Write down differences of characteristics of particles considered under Maxwell-Boltzmann (MB) and Fermi-Dirac (FD) statistics. **[06 marks]**

(i) Consider a system of three particles (A, B and C) each having two possible energy levels, 0 and ε, at absolute temperature T. Indicate possible energy states of the system and total energy of each state if these particles obey MB distribution in a table format. **[05 marks]**

[03 marks]

5.

(a) What is meant by a "Black-body"?

(i) Sketch results of black-body radiation experiment done at temperatures $T_3 > T_2 > T_1$ in a plot of energy vs wavelength. **[03 marks]**

[04 marks]

(ii) Write down three main results that depicts from the above plot. **[05 marks]**

(iii) Discuss how Wein's displacement law can be derived from the plot. **[04 marks]**

(b) State Heisenberg's uncertainty principle in mathematical form, describing each term.

(i) The position and momentum of an electron with energy 0.5 keV are simultaneously determined. If its position is determined with an uncertainty of 0.3 nm, what is the percentage uncertainty of its momentum? **[06 marks]**

6.

(a) Write down the general 3-D expression of time-independent Schrodinger equation for a particle of mass m. Describe what each term represents. **[04 marks]**

(b) Why Schrodinger equation is not valid for relativistic particles? **[03 marks]**

(c) A particle of mass m is confined to a 3-dimentional box of side length l. The potential inside the box is zero.

(i) Obtain expressions for normalized wave function and energy of the particle. **[12 marks]**

$$\left(\text{Hint: } \int_0^l Sin^2\left(\frac{n\pi x}{l}\right) dx = \frac{l}{2}\ for\ n = 1,2,3, \ldots\right)$$

(ii) Write down wave functions and corresponding energies if the particle is found in the ground state and the first excited state. **[04 marks]**

(iii) Discuss degeneracy, if any, in above two cases. **[02 marks]**