UNIVERSITY OF RUHUNA

BACHELOR OF SCIENCE (SPECIAL) DEGREE LEVEL I (SEMESTER II) EXAMINATION – OCTOBER 2021

SUBJECT: Physics

COURSE UNIT: PHY4044 : Electromagnetic Theory **TIME: Three hours**

Answer SIX (06) Questions only

All symbols have their usual meaning.

1. A specified charge density $\sigma(\theta) = \frac{2}{3}\sigma_0 \text{Cos}^2\theta + \frac{1}{3}\sigma_0 \text{Cos}\theta$ is glued over the surface of a spherical shell of radius R. σ_0 is a constant.

   (a) Find the potential both inside and outside of the spherical shell.

   (b) Compute the electric field inside the spherical shell.

2. (a) Point charges are located at the corners of a cube of sides 1 m^2 with one corner placed at the origin and three edges coinciding with the coordinate axes. Values of the point charges in Coulombs 1, -1, 2, -1, 1, 4, -2, and -1 are at the locations (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0), (0, 1, 1), (1, 0, 1), and (1, 1, 1) respectively. Determine the dipole moment of this collection of charges.

   (b) A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a frozen-in polarization, $\vec{P}(r) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the center.

   (i) Determine the bound charge densities and bound charges everywhere.

   (ii) Use the results in part (i) to find the electric field at all points in the space.

   (iii) Find the electric displacement, $\vec{D}$ everywhere.

3. A circular loop of wire, with radius R, lies in the XY plane, centered at the origin, and carries a steady current I running counter clockwise as viewed from the positive Z-axis.

   (a) Determine the magnetic flux density at an axial point at a distance $\mathbf{z}$ from the origin.

   (b) Hence, find the magnetic flux density at the center of the loop.

   (c) Find the approximate vector potential and the magnetic flux density at points far from the origin.

   (d) Show that, for points on the Z-axis, your answer in part (c) is consistent with the exact field as calculated in part (a).

4. Radii of coaxial, perfectly conducting two cylinders are 8 mm and 20 mm. The region between cylinders is filled with a perfect dielectric for which $\varepsilon = \frac{10^{-9}}{4\pi}$ Fm^{-1} and $\mu_r = 1$. If electric field, $\vec{E}$ in this region is $\left(\frac{500}{\rho}\right) Cos(\omega t - 4z)\hat{a}_\rho$ V/m, find followings with the help of Maxwell's equations in cylindrical coordinates.

(a) Angular frequency, ω

(b) Magnetic field intensity, $\vec{H}(\rho, z, t)$

(c) Poynting vector, $\vec{P}(\rho, z, t)$

5. Region 1, where $\mu_{r1} = 4$, is the side of the plane $y + z = 1$ containing the origin. In region 2, $\mu_{r2} = 6$. If $\vec{B}_1 = 2\hat{a}_x + \hat{a}_y$ T, find

(a) the magnetization, $\vec{M}_1$

(b) $\vec{B}_2, \vec{H}_2$ and

(c) the energy density in each region.

6. An air-filled rectangular waveguide with a cross section 4 cm x 7 cm is excited at 3000 MHz.

(a) How many TE modes can the waveguide transmit?

(b) How many TM modes can the waveguide transmit?

(c) Calculate the cutoff frequency of the modes.

(d) Determine all the modes and their cutoff frequency that can be transmitted along the waveguide, if it is excited at 6000 MHz.

7. If a lossless transmission line of characteristic impedance $Z_0 = R_0 = 50\ \Omega$ is terminated in an impedance $Z = (50+25j)\ \Omega$, find

(a) the voltage reflection coefficient

(b) the voltage standing wave ratio and

(c) the input impedance at 0.35λ from the load.

Write down any formula you may use and define each term clearly.

@@@@@@@@@@@@@@@@@@@@

PHY4044 - Equation Sheet

Vector Operations

In Spherical Coordinates

$$\nabla = \frac{\partial}{\partial r}a_r + \frac{1}{r}\frac{\partial}{\partial \theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}a_\phi$$

$$\nabla . A = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla \times A = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\, A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right)a_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)a_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)a_\phi$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

In Cylindrical Coordinates

$$\nabla = \frac{\partial}{\partial\rho}a_\rho + \frac{1}{\rho}\frac{\partial}{\partial\phi}a_\phi + \frac{\partial}{\partial z}a_z$$

$$\nabla . A = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times A = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)a_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)a_\phi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right)a_z$$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}$$

Constants

$\epsilon_0 = 8.85 \times 10^{-12}\frac{F}{m}$ $\mu_0 = 4\pi \times 10^{-7}\frac{H}{m}$ $c = 3 \times 10^8\, m/s$

The Legendre polynomials

$P_0(\cos\theta) = 1$

$P_1(\cos\theta) = \cos\theta$

$P_2(\cos\theta) = \frac{3\cos^2\theta - 1}{2}$

$$\int_0^\pi P_n(\cos\theta)P_m(\cos\theta)\sin\theta\, d\theta = 0 \text{ if } n \neq m$$

$$\int_0^\pi P_n^2(\cos\theta)\sin\theta\, d\theta = \frac{2}{2n+1} \text{ if } n = m$$

Second Derivatives

$\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A$

Product Rules

$\nabla . (A \times B) = B . (\nabla \times A) - A . (\nabla \times B)$

$\nabla . (fA) = f(\nabla . A) + A . (\nabla f)$

$\nabla \times (fA) = f(\nabla \times A) - A \times (\nabla f)$

Orthogonality Property

$$\int_0^\pi \sin mx \sin nx\, dx = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} & m = n \end{cases}$$