UNIVERSITY OF RUHUNA

BACHELOR OF SCIENCE (SPECIAL) DEGREE LEVEL I/II (SEMESTER II) EXAMINATION DECEMBER 2014

Subject: Nuclear and Particle Physics
Course Unit: PHY4084 — Time: Two and half hours

Part II (Essay)

Answer **Five(05)** questions only

All symbols have their usual meaning

Useful constants
Avagadro's number, $A_0 = 6.022 \times 10^{23}$ nuclei/mole
$1\,\text{u} = 931.5\ \text{MeV/c}^2 = 1.66 \times 10^{-27}\,\text{kg}$
Mass of proton, $M_p = 1.007276470\,\text{u}$
Mass of neutron, $M_n = 1.008664904\,\text{u}$
Mass of electron, $M_e = 5.4858026 \times 10^{-4}\,\text{u}$
$\hbar = 1.0546 \times 10^{-34}\,\text{J s}$
$1\,\text{MeV} = 1.602 \times 10^{-13}\,\text{J}$

1. Calculate values for a_c and a_a in the semi-empirical mass formula, using the following facts: $^{35}_{18}Ar$ emits positrons with a maximum kinetic energy of 4.95 MeV and $^{135}_{56}Ba$ is the stable isobar of mass number 135. Express your answer in MeV.
(The semi-empirical mass formula,

$$M(Z,A) = ZM_p + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{A^{3/4}} \text{)}$$

2. A simple quantum mechanical model of the deuteron is based on a spherically symmetric potential well given by

$$\begin{aligned} V(r) &= -V_0 \quad \text{for} \quad r \le R \\ &= 0 \quad \text{for} \quad r \ge R \end{aligned}$$

where V_0 is the depth of the square-well potential and R is a measure of the effective diameter of the deuteron.
The neutron wave functions are: for $r \le R$

$$u_1(r) = A \sin k_1 r + B \cos k_1 r$$

with $k_1 = \sqrt{M(V_0 - E)/\hbar^2}$, and for $r \ge R$

$$u_2(r) = Ce^{-k_2 r} + De^{+k_2 r}$$

with $k_2 = \sqrt{\frac{ME}{\hbar^2}}$.

(a) Use the boundary conditions and the continuity and normalization conditions to evaluate the coefficients A, B, C and D in the neutron wave functions.

(b) If the binding energy of the deuteron is 2.226 MeV, and $R = 2\,\text{fm}$, calculate the depth of the potential, assuming that $V_0 \gg 2.226\,\text{MeV}$ and the mass, $M = 1.667 \times 10^{-27}\,\text{kg}$.

(c) From the resulting wave function, evaluate the root-mean-square radius of the deuteron (you do not need to simplify the expression).

3. The lowest levels in the shell model in order of increasing energy are:

$$1S_{\frac{1}{2}}, 1P_{\frac{3}{2}}, 1P_{\frac{1}{2}}, 1D_{\frac{5}{2}}, 2S_{\frac{1}{2}}, 1D_{\frac{3}{2}}, 1F_j, 2P_j$$

(a) What are the subscript(j) for the lowest $1F$ and $2P$?

(b) What are the configurations of the ground state of the nuclei ${}^{41}_{20}Ca$ and ${}^{41}_{21}Sc$?

(c) What values are predicted in the shell model for their spins, parities and magnetic dipole moments?

(The magnetic moment predicted by the shell model is $\mu = \frac{\mu_N}{2(j+1)}\{g_l[l(l+1) + j(j+1) - s(s+1)] + g_s[s(s+1) + j(j+1) - l(l+1)]\}$. For proton state $g_l = 1$ and $g_s = 5.586$, and for neutron state $g_l = 0$ and $g_s = -3.826$).

4. (a) The fission of ${}^{235}U$ is induced by a neutron and the fission fragments are ${}^{148}La$ and ${}^{87}Br$. Calculate the energy released when 1 g of ${}^{235}U$ fission into ${}^{148}La$ and ${}^{87}Br$. (Atomic masses $M({}^{235}_{92}U) \approx 235.0439\,\text{u}$, $M({}^{148}_{57}La) \approx 147.9320\,\text{u}$, $M({}^{87}_{35}Br) \approx 86.9207\,\text{u}$)

(b) If the efficiency for conversion of heat to electricity is only 5%, calculate the rate of consumption of ${}^{235}U$ fuel per day in a nuclear reactor operating at a power level of 500 MW of electricity.

5. It is expected to operate the proton-proton Large Hadronic Collider at a beam energy of 7 TeV from next year. Rest mass of the proton is $\sim 938\,\text{MeV}$.

(a) Find the total available energy in the center of mass frame of the system for new particle production? If it produces particle-antiparticle pair what would be the maximum limit of the mass of the particle?

(b) Calculate the speed of the proton at this energy.

(c) Calculate the time, with respect to the proton, taken to travel a distance of 3 km at this speed.

(d) Calculate the de Broglie wavelength associated with a proton of the beam. Compare your answer with the size of known particles. ($hc = 12.4 \times 10^{-7}\,\text{eV.m}$)

(e) Suppose such a proton beam is used to collide with another proton at rest in a fixed target experiment. Find(derivation of the relevant expression is expected) the available center of mass energy for particle production in such a fixed target experiment. Compare the importance of collider experiments in compared with fixed target experiments.

(f) Discuss a disadvantage of collider experiment.

6. Explain, qualitatively, the symmetry associated with the conservation of linear momentum, angular momentum, energy and electric charge.
State the relationship between symmetry of the wave-function and statistics obeyed by two identical fermions and bosons respectively.
Particle A decays into two spin half particles. Write down the possible spin wave-function states for the final state two-particle system. Discuss the symmetry properties of the spin wave function.

(a) If the two particles are identical, find the possible values for the spin of particle A.

(b) If the spin of particle A is 1 find the allowed values for the orbital angular momentum of the system of two identical particles.

(c) A heavy meson of spin 1 decays into two pions of zero spin. Show that requirements on the symmetry of the total wave function of the final state prevent the decay into two π^0 mesons. If the meson does decay electromagnetically into $\pi^+\pi^-$, what is the intrinsic parity of the heavy meson?

7. (a) Which of the following reactions are allowed? Verify relevant conservation rules and classify all allowed reactions. Explain each allowed reactions diagrammatically in terms of gauge boson exchange.

   i. $\pi^- + p \to \Sigma^- + K^+$
   ii. $\Lambda_c \to p + k^- + \pi^+$
   iii. $K^- + p \to \Lambda + \pi^0$
   iv. $\pi^- + p \to K^- + K^0$
   v. $K^+ + n \to \Sigma^+ + \pi^0$
   vi. $\Omega^- \to \Lambda + K^-$
   vii. $t \to b + \mu^+ + \nu_\mu$
   viii. $K^+ \to \pi^+ + \pi^0$
   ix. $\mu^+ + \nu_\mu \to e^+ + \nu_e$
   x. $\Sigma^- \to n + \pi^-$

(b) Following reaction shows a possible way of production of Higgs particle (H) and a possible decay channel.

$$p + p \to H + b + \bar{b}$$
$$H \to \tau^+ + \tau^-$$
$$\tau^- \to \mu^- + \bar{\nu}_\mu$$
$$\tau^+ \to e^+ + \nu_e$$

What do you expect to see in the detector? Explain briefly. How would you estimate the mass of the Higgs boson?

8. Answer any three of the following parts

(a) Explain, briefly, the origin of Cosmic Microwave Background Radiation.

(b) Explain the relationship between the range of four fundamental interactions and the mass of their associated gauge bosons.

(c) Why the introduction of colour degree of freedom was necessary? Explain, briefly, considering baryons and mesons.

(d) Explain the properties of particles of baryon decuplet.