University of Ruhuna

B.Sc.(Special) Degree Level I&II(Semester II) Examination
January/February 2013

Subject:Computational Physics
Course Unit: PHY4144 Time: Two hours

Part I (Essay questions)

Answer **four(04)** questions only

All symbols have their usual meaning

1. Given the following equations
(i) $x^4 - x - 10 = 0$ (ii) $x - e^{-x} = 0$

   determine the initial approximations for finding the smallest positive root. Use these to find the root correct to three decimal places with the following methods

   (a) Secant method

   (b) Newton-Raphson method

2. Compute the integral $\int_0^{0.4} \sin(x^2)dx$ correct to three decimal places by repeatedly applying the Trapezoidal rule with $1, 2, 3, 8, \cdots$ panels until the sequence stabilizes

3. Given $\frac{dy}{dx} = 2ty$ and $y(1) = 1$, estimate $y(1.2)$ using step size $h = 0.1$ with

   (a) the Euler method

   (b) the modified Euler method

   (c) the fourth order Runge-Kutta method

4. (a) Use the forward, central, and backward difference formulae to complete the last row of the table

x	0.5	0.6	0.7
$f(x)$	0.493	0.712	0.933
$f'(x)$			

   (b) Using the central difference formula for the second derivative with step size $h = 0.1$, estimate $f''(2)$ when $f(x) = xe^x$. Repeat with $h = 0.2$ and $h = 0.4$ and compare your solutions with the exact answer

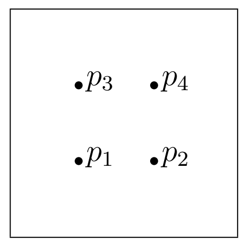
5. (a) Perform Gaussian elimination with pivoting to solve the given system of equations. In each step k, switch rows so as to always pivot using the largest (magnitude) element in column k, that lies in rows k to n

$$\begin{bmatrix} 4 & 1 & -1 \\ 5 & 1 & 2 \\ 6 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$$

(b) Use LU decomposition method to solve the given system of equations below.

$$\begin{bmatrix} 4 & -2 & -1 \\ -2 & 5 & -3 \\ -1 & -3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

6. (a) Determine the system of four equations in the four unknowns p_1, p_2, p_3, and p_4 shown in the figure for computing approximation for the solution $u(x, y)$ to Laplace equation in the square defined by $0 \leq x \leq 3,\ 0 \leq y \leq 3$.



The boundary values are
$U(x, 0) = 10$ and $u(x, 3) = 90$ for $0 < x < 3$
$U(0, y) = 70$ and $u(3, y) = 0$ for $0 < y < 3$

(b) Solve the equations in part(a) for p_1, p_2, p_3, and p_4.